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The dental trauma internet calculator

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Abstract – *Background/Aim*: Prediction tools are increasingly used to inform patients about the future dental health outcome. Advanced statistical methods are required to arrive at unbiased predictions based on follow-up studies. *Materials and methods*: The Internet risk calculator at the Dental Trauma Guide provides prognoses for teeth with traumatic injuries based on the Copenhagen trauma database: http://www.dentaltraumaguide.org The database includes 2191 traumatized permanent teeth from 1282 patients that were treated at the dental trauma unit at the University Hospital in Copenhagen (Denmark) in the period between 1972 and 1991. Subgroup analyses and estimates of event probabilities were based on the Kaplan–Meier and the Aalen–Johansen method. *Results*: The Internet risk calculator shows individualized prognoses for the short- and long-term healing outcome of traumatized teeth with the following injuries: concussion, subluxation, extrusion, lateral luxation, intrusion, avulsion, crown fractures without luxation, root fractures and alveolar fractures. The prognoses for pulp necrosis, pulp canal obliteration, infection-related root resorption, ankylosis, surface resorption, marginal bone loss, and tooth loss were based on the tooth's root development stage and other risk factors at the time of the injury. *Conclusions*: This article explains the database, the functionality and the statistical approach of the Internet risk calculator.

Introduction

The Dental Trauma Guide is one of the world's greatest repositories of knowledge in dental traumatology. It reflects the commonly agreed basics of diagnostics, treatment and prognosis according to the treatment guidelines defined by the international association of dental trauma (IADT) and is constantly updated as new knowledge accumulates. The Dental Trauma Guide can be used by the dental practitioner to verify a diagnosis and to gain valuable advice for treatment and monitoring of the current patient.

The Internet risk calculator is an important part of the Dental Trauma Guide: it opens the door to a large public database on healing events after dental trauma. Here, the dental practitioner is able to extract individualized prognoses for a newly diagnosed traumatized tooth.

Advanced statistical methods and computer algorithms were made available to present the complex internal structure of the database in an understandable format. The results from querying the Internet risk calculator are evidence-based information tailored to the specific characteristics of the newly diagnosed tooth.

The aim of this article is to describe the functionality of the Internet risk calculator, the database, and the statistical approach. An overview is given over the current results that are accessible through the Internet risk calculator.

A clinical case of an 8-year-old girl with an avulsion injury will be used as a practical example of how to use the Internet risk calculator and read the statistical output.

Materials and methods

Data were collected from patients who were treated at the Copenhagen University Hospital in the period between 1972 and 1991 according to a standard protocol and patient records with photo documentation and X-rays.

The construction of the database behind the Internet risk calculator was performed retrospectively in the period between 2005 and 2011. However, the data for all teeth were collected in several prospective clinical studies (1–15), all using the same standardized protocol to document healing complications (1).

For all patients, follow-up examinations at the dental clinic were scheduled after 3 weeks, 6 weeks, 3 or 6 months, 1 year and 5 years after the injury with variation in relation to trauma type. To assess long-term outcome and survival, patients who had at least one tooth with a severe injury type (intrusion or avulsion) were invited to continue in an extended follow-up schedule with an examination at the dental clinic after 10 years, in some cases also after 20 years after the injury.

Inclusion criteria

1 Standardized protocols including radiographs were available from the time of injury and from at least one subsequent control later than 10 months after the injury.
2 Clinical photographs were available from the time of injury.

3 The patient had no history of previous trauma affecting the permanent incisors.
4 The tooth had no large destruction of the crown caused by dental caries or restorations.

Statistical methods

Failure time analysis was performed to predict healing complications (events) based on the data available at the time of injury. Exact onset times of healing complications were not available (interval censored), and all analyses were based on approximate event onset times. For each healing complication, an approximate onset time was defined as the midpoint between the examination time of the first examination where the event was diagnosed and the examination time of the previous examination. In cases where two or more healing complications were diagnosed at the same examination, approximate event times were distributed in biological order. If a certain healing complication was not diagnosed at any follow-up examination, the approximate event time was right censored at the latest examination time. The prediction of risk was based on the Kaplan–Meier method for events in the absence of competing risks and the Aalen–Johansen method for events in the presence of competing risks (16).

Pointwise confidence limits were constructed using a formula for the variance of the Kaplan–Meier estimate which is suitable for dental applications where some of the event times belong to the same patient and hence are dependent (17–20). For the Aalen–Johansen estimate, such a robust variance formula was not available and the standard Greenwood variance formula was used (16). For both methods, in cases where zero healing complications were recorded in our data, pointwise confidence limits for the risk at a specific time point were based on the current number of teeth in study and on the exact binomial distribution.

Results

The database behind the Internet risk calculator contains records from 2191 permanent teeth with a traumatic injury belonging to 1282 different patients.

Patient characteristics

Data were collected from 456 women and 826 men aged between 5 years and 81 years at the day of injury with one or several injured teeth (Table 1). About half of the patients, $n = 637$ (49.7%), had only one injured tooth, about one-third of the patients, $n = 445$ (34.7%) had two injured teeth, further $n = 152$ patients (11.9%) had three injured teeth and $n = 48$ patients (3.7%) had more than three and up to seven injured teeth.

Follow up

One year after the injury, the full information on survival status and healing complications was available for $n = 1857$ teeth (84.8%) of the $n = 2191$ teeth included in the database. Follow up was longer than 3 years after the injury for $n = 1025$ teeth (46.8%), longer than 5 years after the injury for $n = 744$ teeth (34.0%), and longer than 10 years after the injury for $n = 307$ teeth (14.0%). Long-term information was available for $n = 101$ teeth (4.6%) with more than 15 years of follow up and $n = 30$ (1.4%) with more than 20 years of follow up.

Table 1. Patient characteristics

Age in years	Number of patients	Female (%)	Male (%)
2–6	59	27 (45.76)	32 (54.24)
6–8	297	116 (39.06)	181 (60.94)
8–10	238	80 (33.61)	158 (66.39)
10–12	137	53 (38.69)	84 (61.31)
12–15	143	43 (30.07)	100 (69.93)
15–18	123	31 (25.2)	92 (74.8)
18–21	87	21 (24.14)	66 (75.86)
21–25	71	27 (38.03)	44 (61.97)
25–30	40	15 (37.5)	25 (62.5)
30–40	48	21 (43.75)	27 (56.25)
40–60	32	16 (50)	16 (50)
60–81	7	6 (85.71)	1 (14.29)
Total	1282	456 (35.57)	826 (64.43)

Typology of dental trauma injuries

Teeth were classified at the initial examination according to one of the following main injury types:

concussion, subluxation, extrusion, lateral luxation, intrusion, avulsion, crown-root fracture without pulp involvement, crown-root fracture with pulp involvement, root fracture, and alveolar fracture (Table 2). All teeth with alveolar fractures were classified as alveolar fractures even teeth that had other concomitant injuries (root fractures and crown-root fractures).

Crown fractures that were not part of a combination injury were classified as one of the following crown fracture types: infraction, enamel fracture, enamel-dentin fracture, enamel–dentin–pulp fracture.

Table 2. Distribution of injury types in the dental trauma database according to the root development stage at the time of the injury

Injury type	Number of teeth	Immature root development (%)	Mature root development (%)
Concussion	469	169 (36.03)	300 (63.97)
Subluxation	404	230 (56.93)	174 (43.07)
Extrusion	83	43 (51.81)	40 (48.19)
Lateral luxation	181	50 (27.62)	131 (72.38)
Intrusion	141	52 (36.88)	89 (63.12)
Avulsion	400	110 (27.5)	290 (72.5)
Crown fractures[1]	238	81 (34.03)	157 (65.97)
Root fracture	88	13 (14.77)	75 (85.23)
Alveolar fracture	187	7 (3.74)	180 (96.26)
Total	2191	755 (34.46)	1436 (65.54)

[1]crown fractures without luxation injury.

Table 3. Crude number of healing complications by injury type that were diagnosed during follow-up and recorded in the database

	Pulp necrosis	Prophylactic pulp extirpation	Pulp canal obliteration	Repair-related resorption
Concussion	22	0	16	12
Subluxation	54	0	37	6
Extrusion	29	0	32	8
Lateral luxation	103	0	48	38
Intrusion	125	0	16	3
Avulsion	368	290	32	16
Crown fracture[1]	5	0	2	0
Root fracture	22	0	63	0
Alveolar fracture	77	16	24	10
	Infection-related resorption	Ankylosis	Marginal bone loss	Tooth loss
Concussion	0	0	2	0
Subluxation	5	1	3	0
Extrusion	3	0	7	0
Lateral luxation	4	1	12	0
Intrusion	38	40	45	28
Avulsion	119	255	29	118
Crown fracture[1]	0	0	0	0
Root fracture	0	1	5	9
Alveolar fracture	5	3	13	10

Predictor variables

For different trauma injury types, different characteristics of the teeth were considered as prognostic factors for the healing outcomes. The injury type was considered the most important predictor for the healing outcome. For each injury type separately, specific prognostic variables were selected of the patient and the injured tooth to refine the prognosis. This selection was primarily carried out with respect to the available factors that were known to have an effect on the healing outcome. However, in several cases, prognostic variables could not be used to further discriminate teeth because of limitations of the database (too few teeth in subgroups). For all luxation injury types, the root development stage ('immature': stages 1–5, 'mature': stage 6) was used. For teeth with alveolar fracture, this was the only factor. For teeth with root fracture, the root fracture position was the only additional factor with three levels (apical, coronal, and mid). Teeth with avulsion injury were further subdivided according to the dry extra alveolar time with three levels (0–4 min, 5–60 min, more than 60 min) and according to the wet extra alveolar time in physiologic media with 2 levels 0–4 min, more than 5 min. Teeth without luxation injury were subdivided according to the crown fracture type with four levels (infraction, enamel fracture, enamel-dentin fracture, and enamel–dentin–pulp fracture). Teeth with concussion, subluxation, extrusion, lateral luxation, and intrusion injury were subdivided into two groups based on whether they had a concomitant crown fracture or not. Finally, teeth with concussion and subluxation were optionally subdivided according to the results of the electric pulp test at the initial examination.

Healing outcome-related events

The following healing complications were recorded in the standardized protocol and considered for statistical evaluation: pulp canal obliteration, pulp necrosis, repair-related root resorption, infection-related root resorption, ankylosis, marginal bone loss, and tooth loss (Table 3).

General usage

The Internet risk calculator provides information about what happened to the teeth of former patients with a similar traumatic tooth injury as the presenting new patient. To start the tool, the first step is to choose the luxation injury type. In the next level, the stage of the root development is specified. The tool distinguishes in all cases between mature teeth with fully developed root (stage 6) and immature teeth (stages 1–5) for the prognosis. It then depends mostly on the injury type whether or not further information regarding the injured tooth can be used to refine the prognosis (see predictor variables).

At the press of the button, a series of graphs and tables appears. These contain the prognosis tailored to the specified tooth and its trauma. In separate tables, the risks of the healing complications are shown for the periods until 1, 3, and 10 years after the injury. The risks are shown on a probability scale between 0% and 100% and supplied with time pointwise 95% confidence limits. The graphs show the time development of the risks of healing complications as increasing functions on a probability scale between 0% and 100% (red lines) with pointwise 95% confidence limits (pink shadows).

The user can manually switch between the predicted risks of the different healing complications and tooth loss, and between three different time horizons (1, 3, and 10 years).

Predictions for a tooth with intrusion injury

To illustrate how the Internet risk calculator can guide decision making, consider a hypothetical newly traumatized tooth with an intrusion injury. Suppose further that the tooth has a fully developed root (stage 6) and has an enamel-dentin crown fracture. Note that for this injury type, the Internet risk calculator distinguishes not between the four different crown fracture types (infraction, enamel fracture, enamel-dentin fracture, and enamel–dentin–pulp fracture).

After choosing 'Intrusion', move on to the prognosis section. There is now a choice between different stages of root development and different types of crown fracture. Mature root development and enamel-dentin fracture is marked and the calculator shows that 'the data base included 68 teeth with the same or a similar diagnosis as yours. These teeth belong to 45 patients.' (Fig. 1).

The risks of the different healing complications are shown using graphical display and tables. For example, Fig. 2 shows the risk of ankylosis resorption for a 10-year time horizon.

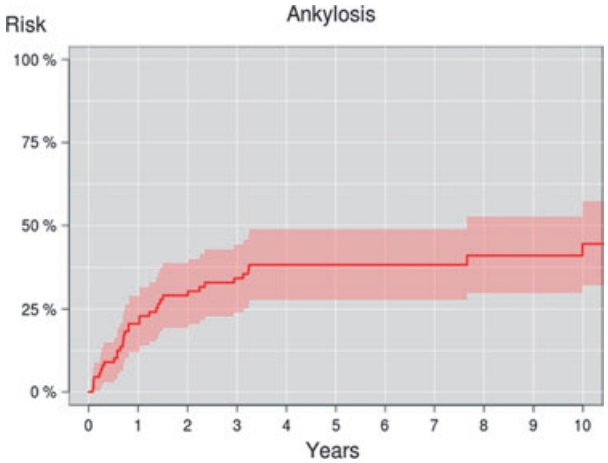

Fig. 2. The Dental Trauma Internet Calculator: Predicted risk of an intruded tooth with mature root development. The prediction is based on what happened to 68 teeth in 45 patients from the database that had similar injury. The red line shows the estimated risk of ankylosis resorption estimated by the Aalen–Johansen method, and the pink shadow time pointwise 95% confidence limits.

After 1 year, the predicted risk of ankylosis resorption is roughly 25%. The estimated risk is further increasing to about 45% after 10 years. The curves for the ankylosis resorption risk were obtained with the

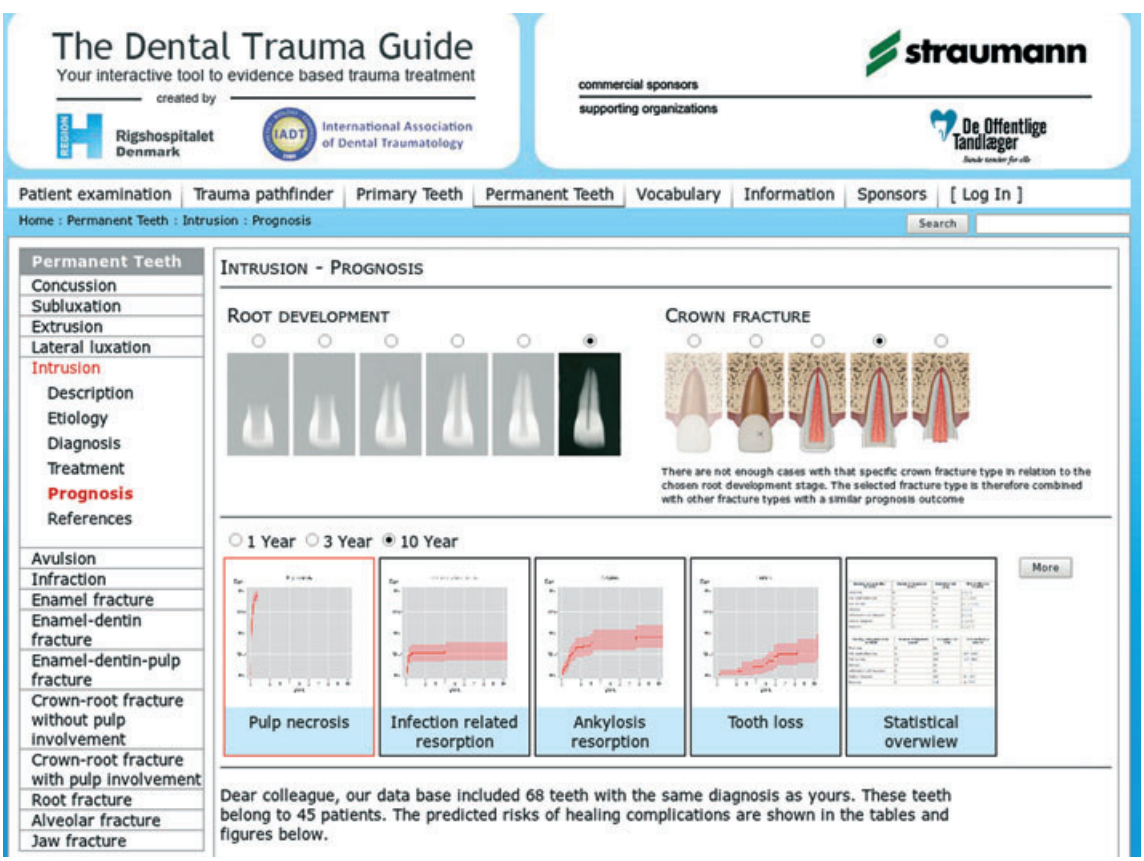

Fig. 1. The Dental Trauma Internet Calculator: Prognosis for an intruded tooth with mature root development and crown fracture.

Table 4. Statistical overview for intrusions with mature root development and a concomitant crown fracture. The table gives and overview of predicted risks (in%) for a set of healing complications after 1, 3, and 10 years after the injury with 95% confidence limits. If not enough teeth were followed, confidence intervals were not available and the results should not be interpreted

Results	Number of diagnosed events	Estimated risk (%)	95% confidence interval
One year after accident			
Tooth loss	2	2.9	[0–7]
Pulp necrosis	66	57.1	[93–100]
Pulp canal obliteration	1	1.5	[0–4.3]
Ankylosis	I6	23.5	[13.4–33.6]
Inflammatory root resorption	18	26.5	[16–37]
Surface resorption	1	1.5	[0–4.3]
Bone loss	22	32.4	[21.2–43.5]
Three years after accident			
Tooth loss	3	8.1	[1.3–14.9]
Pulp necrosis	1	98.5	[95.7–100]
Pulp canal obliteration	1	0	not available
Ankylosis	8	36.5	[24.8–48.3]
Inflammatory root resorption	0	26.5	[16–37]
Surface resorption	0	1.5	[0–4.3]
Bone loss	3	36.9	[25.4–48.4]
Ten years after accident			
Tooth loss	8	26.5	[14.6–42.4]
Pulp necrosis	0	NA	not available
Pulp canal obliteration	1	0	not available
Ankylosis	4	45.6	[32.2–59.1]
Inflammatory root resorption	1	28.6	[17.6–39.6]
Surface resorption	0	1.5	[0–4.3]
Bone loss	5	45.8	[33.5–58]

Aalen–Johansen method (16,17) taking into account that tooth loss is a competing risk. For a tooth with this injury, the risk of tooth loss during the first 3 years after the injury is 8.1% (see Table 4). The table shows that most likely a tooth with this injury will develop pulp necrosis within the first 3 years after the injury (predicted risk: 98.5%).

Discussion

The Dental Trauma Guide is structured into sections dealing with each of the individual dental trauma injuries separately. The Internet risk calculator presented here provides the results from statistical analyses for each of the dental trauma injury types on separate pages. The different dental trauma injury types have different biologic healing conditions and it was therefore decided to analyze the different injury types separately.

The four crown fracture types (infraction, enamel fracture, enamel-dentin fracture, and enamel–dentin–pulp fracture) were grouped for teeth with concussion and subluxation. The prognosis for teeth with enamel–dentin–pulp fractures is probably more sensitive to the choice of treatment and treatment delay than the other fracture types, and the prognosis provided by the Internet risk calculator is therefore only meaningful for teeth treated under similar conditions. Crown fracture with pulp exposure occurred very rarely in combination with extrusion and lateral luxation, and a prognosis for this type of combination injury based on our data is therefore not available.

Treatment

All teeth that received prophylactic pulp extirpation treatment have been excluded from the analysis of prognosis for pulp healing, but have been included in the analysis of prognosis for root resorptions, marginal bone loss and tooth loss.

All teeth diagnosed with an avulsion injury that also had mature root formation received prophylactic endodontic treatment.

The reason for this approach was that healing of the pulp without treatment was assessed as unlikely. By removing the nonvital tissue from the root canal, the imminent risk of infection-related root resorption was minimized. An implication of this decision is that our database cannot be used to predict the pulp healing outcome for teeth with avulsion and mature root formation. Also part of the teeth diagnosed with alveolar fracture received prophylactic endodontic treatment to avoid pulp necrosis. Those teeth were treated in a period where the fairly good chances of pulp healing following alveolar fractures were still unknown.

Competing risks

It was important to note that some of the healing complications are mutually exclusive. For example, pulp canal obliteration can only occur before the onset of pulp necrosis and infections-related root resorption may only occur after the onset of pulp necrosis. We applied appropriate statistical models (competing risk models) which take these biological constrains into account, and obviously also that no healing complication can occur after tooth loss.

Confidence limits

In some specific subgroups, no healing complications were observed. The usual construction of time pointwise confidence intervals fails when there are no events recorded. In the absence of more appropriate statistical tools, we decided for an ad hoc solution and computed time pointwise confidence limits based on the current number of teeth in study from the exact binomial distribution.

For the Aalen–Johansen estimate, a corresponding variance formula and software does not seem to exist, which would take into account the clustered data structure (multiple injured teeth in the same mouth). The bootstrap method could not be applied because of the

small sample sizes. We have therefore constructed time pointwise confidence limits based on the standard Greenwood variance formula.

How should the Internet risk calculator on the Dental Trauma Guide be used?

In the following, a practical example of how to use the Internet risk calculator will be illustrated by a clinical case.

The patient is a 8-year-old girl, who has avulsed her left central incisor. The extra alveolar dry time was 3 min and the tooth was kept wet in the oral cavity (saliva) for 30 min. On radiographic examination, of the injured central incisor, the root development stage was found to be stage 3 (3/4 root length and width open apex). The tooth had been stored in a physiologic media and hence meets the requirements for using the Internet risk calculator. The risk prognosis was based on the stage of root development (stage 3), the length of extraoral dry (1–4 min), and wet time (5- min).

The Internet risk calculator found 19 teeth from 16 patients who met the same criteria as the one of our case study. Figure 3 shows the results of the pulp necrosis risk analysis.

The figure shows that the risk of pulp necrosis after 3 years is high 57.9% (95% CI: 35.7%–80.1%) and that it is likely that an eventual pulp necrosis can be positively verified within the first 3 months (steep increase of the curve).

The graph for pulp necrosis is discontinued after 3 years as the risk calculator is not able to give any further information about the risk of pulp necrosis after this point. The reason is that not enough (<3 teeth) of the 19 teeth for which pulp necrosis was not diagnosed until 3 years was examined at a later time point. Note that in the Internet risk calculator the graphs for any given healing complication is terminated when only 3 patients are still at risk for that particular healing complication. It is apparent that graphs for other healing complications and tooth loss continue past the 3-year mark. The reason is that some of the teeth that were diagnosed with pulp necrosis during the first 3 years were examined at later time points.

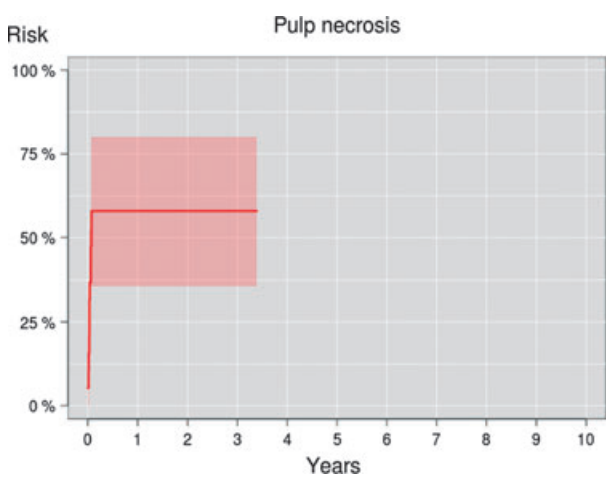

Fig. 3. Sample case study: Predicted risks of an avulsed tooth with immature root development stage, short dry time (0–4 min), and long wet time (5+ min). The red line shows the estimated risk of pulp necrosis and the pink shadow time pointwise 95% confidence limits.

The risk of infection related resorption after 3 years was relatively high (36.8% (95% CI: 15.2%–58.5%)). It is therefore important to monitor a patient like the presently considered closely. The risk of ankylosis after 3 years was 36.8% (95% CI: 15.2%–58.5%).

The risk of tooth loss after 10 years was quite high 40.1% (95% CI: 8.7%–71.6%) which emphasizes the need for long-term treatment planning. However, the large confidence limits indicate that the statistical uncertainty about the precise risk of tooth loss after 10 years is quite high.

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