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Three-dimensional analysis of orthodontic tooth movement based on *XYZ* and finite helical axis systems

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SUMMARY The purpose of this study was to demonstrate the advantage of the finite helical axis (FHA) system in the biomechanical analysis of orthodontic tooth movement by comparing it with the rectangular coordinate (*XYZ*) system.

Ten patients (6 females and 4 males, mean age 23 years 7 months) were selected. Maxillary canine retraction using light continuous forces of two different magnitudes (0.5 and 1 N) was used to retract the right and left maxillary canines in subjects who required maximum posterior anchorage. The findings were compared based on midpalatal implants that provided a fixed reference for measurement. The significance of the difference between the results with the two different force magnitudes was determined using Wilcoxon's signed-rank test.

With both the *XYZ* and the FHA system, no significant differences in the amount of distal movement of the canines over 2 months were found between the two force magnitudes. However, the results showed that the canine was likely to incline distally during tooth retraction with a force of 1 N compared with a force of 0.5 N ($P < 0.05$). With the FHA system, the result indicated that the canine was likely to incline palatally during tooth retraction with a force of 0.5 N compared with a force of 1 N ($P < 0.05$).

In this study, the combination of these two different approaches for describing tooth movement clearly showed a difference between light continuous forces of 0.5 and 1 N.

Introduction

An accurate understanding of orthodontic tooth movement resulting from applied mechanics is necessary for elucidating the optimum force magnitude and the type of system that should be used. Many investigators have focused on studying the optimum force magnitude for orthodontic tooth movement. Unfortunately, previous investigations on the magnitude of force or stress and the rate of tooth movement have not shown a strong positive relationship in humans or animals (Hixon *et al.*, 1969; Boester and Johnston, 1974; Andreasen and Zwanziger, 1980; Owman-Moll *et al.*, 1996; Pilon *et al.*, 1996). A systematic review of the literature showed that no evidence-based force level could be recommended to achieve optimal efficiency in clinical orthodontics (Ren *et al.*, 2003). One of the major complicating factors is the difficulty of representing three-dimensional (3D) tooth movement.

In orthodontics, it is important to determine what type of movement has occurred. In a planar environment, the type of movement can be analysed using the relationship between the centre of resistance (CRes) and the centre of rotation (CRot; Smith and Burstone, 1984). The distance from the CRes to the CRot increases in proportion to the translation (bodily movement) of a tooth. Even though orthodontic tooth movement is generally 3D, a consideration of planar movement using the finite CRot is still an effective approach for clinical use. However, more detailed information on tooth movement through space is needed for biomechanical analysis.

Several morphometric and theoretical studies have been conducted based on a six-degrees-of-freedom rectangular coordinate (*XYZ*) system (Iwasaki *et al.*, 2000; Ashmore *et al.*, 2002; Hayashi *et al.*, 2004a). However, this representation varies depending on the position of the coordinate origin, the rotation sequence of the axis, and the timing of translation (Richard, 1984). In particular, the reliability of computing rotation based on the *XYZ* system is poor. In one report, the average rotation ranged from –2.48 to 1.31 degrees (Ashmore *et al.*, 2002). A finite helical axis (FHA) can be used to solve the problems of superscription, and has been applied to the analysis of joint function (Spoor and Veldpaus, 1980; Woltring *et al.*, 1985; Gallo *et al.*, 1997, 2000; DeLong *et al.*, 2002). A new method for calculating the FHA has previously been reported and applied to the 3D analysis of orthodontic tooth movement (Hayashi *et al.*, 2002, 2004b, 2005, 2006). An analysis of the direction vector of the FHA could provide more accurate information about tooth movement for biomechanical analysis than the *XYZ* system since the values of the parameters of the FHA do not change with coordinate transformations. Thus, the FHA parameters are the same for all points on a moving body at any particular time point, and this may lead to improved understanding of how to move teeth.

The purpose of this study was to demonstrate the advantage of the FHA in the biomechanical analysis of orthodontic tooth movement. Maxillary canine retraction using forces of different magnitudes was compared based on midpalatal orthodontic implants, which provided a fixed reference for measurement. Light continuous forces of two different magnitudes were used to retract the right and left maxillary canines in subjects who required maximum posterior anchorage. To obtain more detailed and accurate information, a 3D analysis of tooth movement was conducted using not only the *XYZ* system but also the FHA system.

Subjects and methods

Subjects

Ten patients (6 females and 4 males, mean age 23 years 7 months, range 19 years 4 months to 29 years 2 months) from the orthodontic clinic of Health Sciences University of Hokkaido were selected. All were informed of the experimental protocols and signed an informed consent form approved by the Institutional Review Board.

Since maximum posterior anchorage was required in all subjects, osseointegrated midpalatal implants (Institute Straumann AG, Waldenburg, Switzerland) were used. After a 3-month healing period following implant placement, impressions were obtained using a conventional technique to transfer the impression post (molar bands if necessary) to a dental cast. A 1.2 mm^2 stainless steel rigid wire was soldered to mesh plates on the molar palatal surface or to molar bands and connected to the implant abutment on the casts. As a registration reference on dental casts for 3D tooth movement analysis, three markers (steel ball bearings; φ 2.778 mm) were soldered to the rigid wire (Figure 1a). After the laboratory procedures, rigid wire was fixed to the palatal implant. Orthodontic bands were cemented and mesh plates were bonded to the first molars. In all subjects, the maxillary first premolars were extracted as part of the treatment plan. Canine retraction was commenced approximately 4 months after implant placement.

Canine retraction mechanics

The labial arch was made of 0.45 mm (0.018 inch) stainless steel wire and included the second molar, first molar, second premolar, canine, and incisors. Standard edgewise brackets (Micro-LOC, Tomy International Inc., Tokyo, Japan) with a 0.022 × 0.028 inch slot were used. Two different retraction forces were produced using nickel-titanium (NiTi) closing coil springs (Sentalloy, Tomy International Inc.). The Sentalloy closing coil springs were extra light- (black) and light- (blue) grade, which the manufacturer stated provided forces of 0.5 and 1 N when stretched within a range of 3–15 mm, respectively. To confirm this characteristic, each closing coil spring was measured with a dial tension gauge (Mitutoyo, Kawasaki, Japan). The NiTi closing coil springs were engaged between the first molar tube hook and the sliding hook placed mesially on the canine bracket. A figure-of-eight ligation was used to increase anchorage of the posterior segment. In all subjects, a continuous force of 0.5 N was applied to the right maxillary canine, and 1 N to the opposite canine. The NiTi closing coil springs were stretched to a length of 12 mm weekly (Figure 1b).

Analysis of dental casts

To examine tooth movement, impressions of the maxillary arch were made at each appointment with hydrophilic vinyl polysiloxane impression material (JM Silicone, J. Morita, Tokyo, Japan) using customized acrylic impression trays lined with adhesive, and poured in a die stone (Noritake super rock, J. Morita). A 3D surface-scanning system using a slit laser beam (VMS-150RD, Unisn, Osaka, Japan) was used to measure the series of dental casts. The system consisted of a slit laser projector, two charge-coupled device cameras, an auto-rotating mounting unit, and a personal computer with post-processing software. The accuracy of this measuring device has been reported previously (Hayashi *et al.*, 2002): the resolution in the *X*-direction was 0.01 mm, and the *Z*-direction could be measured to within ±0.05 mm. The 3D shape data-analysis system consisted of a graphic workstation (Zx1, Intergraph, Huntsville, Alabama, USA) and data-processing and analysing software (I-DEAS, SDRC, Milford, Connecticut, USA). To estimate 3D tooth movement, a common coordinate system was established. The baseline model data (before canine movement) were orientated in a world coordinate system based on the three fixed markers and anatomic structures. Each subject's subsequent models

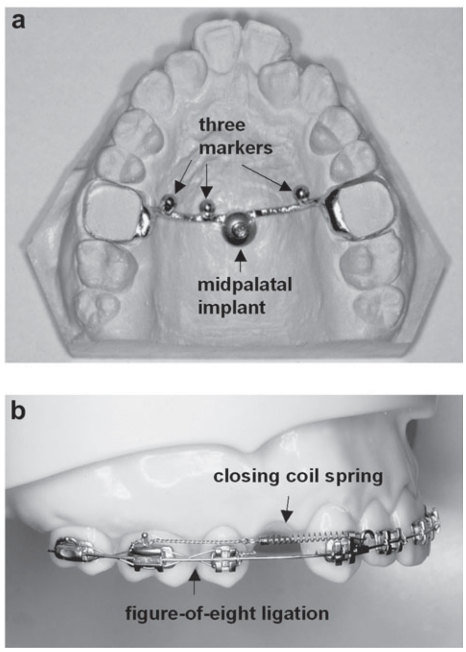

Figure 1 (a) A midpalatal orthodontic implant and three markers soldered to the rigid wire as a registration reference on the dental cast. The three markers were placed to make a triangle as large as possible. (b) The sliding mechanics used in this study. The closing coil springs were extra light- (black) and light- (blue) grade, which the manufacturer reported provided forces of 0.5 and 1 N when stretched within a range of 3–15 mm, respectively.

were superimposed on the baseline model by the least-squares method using the three markers as registration structures, which were considered to be stable during treatment (a process called surface registration). To minimize registration error, the clamping cap of the osseointegrated implant was also used as a registration structure because of the possibility of deflection of the palatal bar (Figure 2). The discrepancy between images of the maxillary canine was recognized as tooth movement. A space coordinate system was established based on the anatomic shape of the clinical crown (Figure 2) and corresponded to the world coordinate system at baseline. The movement of an individual tooth could be expressed by a translation vector and a rotation matrix obtained by automatic surface registration of the 3D shapes before and after tooth movement (Hayashi *et al.*, 2002).

Tooth movement: XYZ and FHA systems

To represent the change in the position of a canine based on the *XYZ* system, six parameters were introduced. These were defined by the translations Δx, Δy, and Δz, and the rotations ψ, θ, and φ, which denote the tipping, flaring, and rotation of the tooth coordinate system, respectively (Figure 3a). In this study, the sequence of rotation in the *XYZ* system was in the order *X*, *Y*, and *Z*.

A new method for calculating the FHA has previously been reported (Hayashi *et al.*, 2002), which can be summarized as follows. Each point $P'(x', y', z')$ on the 3D shape after movement was expressed in the *XYZ* system as a function of the six parameters that describe translation and rotation. Thus, the 3D displacement of $P(x, y, z)$ on the 3D shape before movement was determined from the formula

$$a = Ab + c, \tag{1}$$

where A is a 3×3 rotation matrix, $\boldsymbol{a}$ is the position vector of arbitrary points after tooth movement, $\boldsymbol{b}$ is the position vector of arbitrary points before tooth movement, and $\boldsymbol{c}$ is the translation vector. Tooth movement in 3D space was then determined as a displacement vector $\boldsymbol{r}$ with components $(x' - x)$, $(y' - y)$, and $(z' - z)$. This movement involves both translation and rotation. The displacement vector $\boldsymbol{r}$ was expressed as

$$\boldsymbol{r} = (A - E)\boldsymbol{b} + \boldsymbol{c}, \tag{2}$$

where E is a unit matrix, which is a function of the coordinates before movement. The 3×3 matrix A reflects rotation about the axis which passes through the origin and parallel to the FHA. For points on the FHA, the displacement vector $\boldsymbol{r}$ is parallel to this axis and does not change with rotation, so the axis position follows from

$$\boldsymbol{r} = A^T \boldsymbol{r}. \tag{3}$$

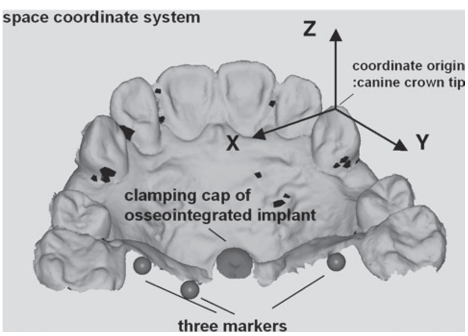

Figure 2 Three-dimensional images of a dental cast at baseline. A space coordinate system was established based on the anatomical shape of the clinical crown.

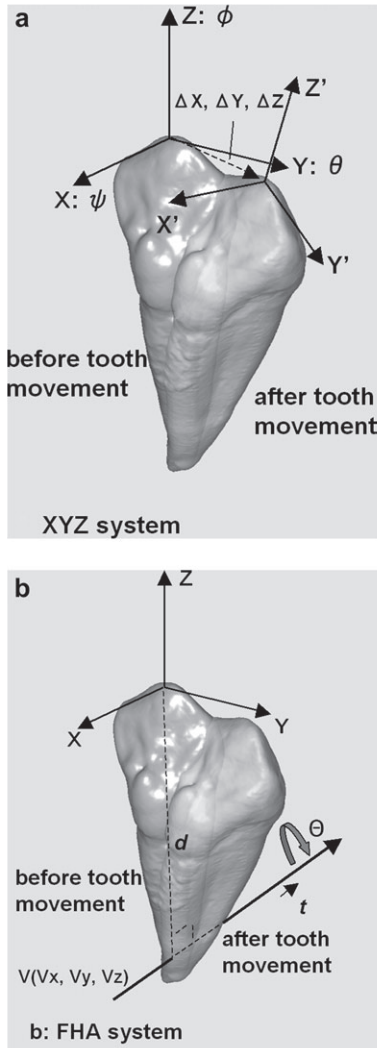

Figure 3 (a) Coordinate system for determining canine retraction using the *XYZ* system. Translations are denoted by Δx, Δy, and Δz. and rotations by ψ, θ, and φ, which represent the tipping, flaring, and rotation angles, respectively. (b) Calculated finite helical axis (FHA) parameters: the direction vector of the FHA $\boldsymbol{v}$ (v_x, v_y, v_z), the rotation angle about the FHA θ, the translation along the FHA $\boldsymbol{t}$, and the shortest distance from the space coordinate origin (canine crown tip) to the FHA $\boldsymbol{d}$.

Therefore, by multiplying equation (2) by A^T and applying the relationship in equation (3),

$$(A-E)\boldsymbol{b}+\boldsymbol{c}=A^{\mathrm{T}}(A-E)\boldsymbol{b}+A^{\mathrm{T}}\boldsymbol{c}; \tag{4}$$

equation (4) can be written as

$$(A+A^{\mathrm{T}}-2E)\boldsymbol{b}+(E-A^{\mathrm{T}})\boldsymbol{c}=\boldsymbol{0}. \tag{5}$$

Equation (5) also follows from

$$\frac{d}{d\boldsymbol{b}\{((A-E)\boldsymbol{b}+\boldsymbol{c})^{\mathrm{T}}((A-E)\boldsymbol{b}+\boldsymbol{c})\}}=\boldsymbol{0}. \tag{6}$$

which gives the points $\boldsymbol{b}$ that have minimal displacements and therefore lie on the FHA. The relationship in equation (5) is valid at all points on the FHA. While the solution is not unique, it can be obtained by substituting a value for the *Z*-component.

When $\boldsymbol{b}$ is chosen on the FHA, $\boldsymbol{a}$, resulting from equation (1), it is on the same axis. The distance between $\boldsymbol{a}$ and $\boldsymbol{b}$ is a translation, $\boldsymbol{t}$, along the FHA. Next, an arbitrary point, $\boldsymbol{d}_0$, outside of the FHA was chosen, and its position after tooth movement $\boldsymbol{d}_1$ was calculated again by equation (1):

$$\boldsymbol{d_1}=A\,\boldsymbol{d_0}+\boldsymbol{c} \tag{7}$$

Next, the orthogonal projection of $\boldsymbol{d}_0$ and $\boldsymbol{d}_2$ ($\boldsymbol{d}_2 = \boldsymbol{d}_1 - \boldsymbol{t}$) on the FHA was calculated:

$$\boldsymbol{q}'=\boldsymbol{q}+\frac{(\boldsymbol{d}-\boldsymbol{q})\times\boldsymbol{l}}{\boldsymbol{l}\times\boldsymbol{l}}\boldsymbol{l}, \tag{8}$$

where $\boldsymbol{q}'$ is the position vector of a point on the FHA, $\boldsymbol{q}$ is the position vector of an arbitrary point on the FHA, $\boldsymbol{d}$ is $\boldsymbol{d}_0$ or $\boldsymbol{d}_2$, and $\boldsymbol{l}$ is the direction of the FHA. Thus, the rotation angle θ about the FHA in space is identical to the angle between $\boldsymbol{d}_0$, $\boldsymbol{q}'$, and $\boldsymbol{d}_2$. The calculated FHA parameters were the direction vector of the FHA $\boldsymbol{v}$ (v_x, v_y, v_z), the rotation angle about the FHA θ, the translation along the FHA $\boldsymbol{t}$, and the shortest distance from the tooth coordinate origin to the FHA $\boldsymbol{d}$ (Figure 3b).

Parameters for the *XYZ* system were determined weekly for 2 months. Parameters for the FHA system were determined only at baseline and 2 months because the canine positions can clearly demonstrate the mechanical differences between two forces (Hayashi *et al.*, 2006). Tipping, flaring, and rotation angles in the *XYZ* system could be compared with the direction vector of the FHA weekly for 2 months. However, the direction vector of the FHA tends to change significantly from week to week. Although this means that the biomechanical forces are not uniform during the treatment period, it is difficult to compare with the *XYZ* system without a collinear approximation or modelling of a spline function. Therefore, for simplicity, only the parameters for the FHA system that were determined from canine positions at baseline and 2 months were used.

The significance of the difference between the results with the two different force magnitudes was determined using Wilcoxon's signed-rank test.

Results

XYZ system

The amount of distal movement of the canine crown tip with a light continuous force of 0.5 N was 3.16 mm over 2 months (Table 1). With a force of 1 N, this value was 3.39 mm. These values were not significantly different. Figure 4a shows the average movement profiles over time for all teeth subjected to both forces. Plots of average cumulative distal movement showed that no lag phase was associated with maxillary canines moved by 0.5 N. However, with 1 N, a distinct lag phase was evident at week 2.

The canine tipping values over 2 months were 6.99 degrees with 0.5 N and 8.22 degrees with 1 N. These values were significantly different ($P < 0.05$). Figure 4b depicts the average tipping profiles over time for all teeth subjected to both forces. The canine tipping angles were not increased at 2 weeks with 1 N. These results demonstrated that the observed lag phase was caused by absence of tipping movement.

The canine flaring values with a force of 1 N over 2 months were –1.39 degrees and with 0.5 N –1.54 degrees. A negative value for canine flaring indicated palatal inclination of the tooth. The canine rotation values over 2 months were 4.11 degrees with 0.5 N and 4.30 degrees with 1 N. These values were not significantly different. Figure 4c,d shows the average flaring and rotation profiles over time for all teeth subjected to both forces.

FHA system

The amount of distal movement of the canine crown tip with a light continuous force of 0.5 N calculated using the FHA system was 3.17 mm over 2 months (Table 2). With 1 N, this value was 3.38 mm. These values were almost the same as those with the *XYZ* system, and were not significantly

Table 1 *XYZ* system parameters: the amount of distal movement (mm), tipping, flaring, and rotation (degrees) of a canine over a 2-month treatment period.

XYZ system parameters	0.5 N		1 N		*P*	
	Mean	SD	Mean	SD		
Distal movement of canine crown tip (mm)	3.16	0.60	3.39	0.70	0.236	NS
Tipping angle ψ of canine (degrees)	6.99	2.10	8.22	2.23	0.048	*
Flaring angle θ	−1.39	2.12	−1.54	1.22	0.583	NS
Rotation angle φ of canine (degrees)	4.11	4.79	4.30	7.62	0.824	NS

*$P < 0.05$. NS, not significant.

different. The FHA yields a comprehensive 3D description of rigid body motion. Rotation around the FHA θ is equivalent to a combination of three rotations in the *XYZ* system (tipping, flaring, and rotation). In this study, the amount of rotation around the FHA θ over 2 months was 12.24 degrees with 0.5 N and 12.91 degrees with 1 N. These values were not significantly different. True translational movement of a rigid body can be determined by analysing the translation along the FHA $\boldsymbol{t}$ and the shortest distance from a tooth to the FHA $\boldsymbol{d}$. On the other hand, an increase in the extent of bodily tooth movement strongly influences both $\boldsymbol{t}$ and $\boldsymbol{d}$. However, the $\boldsymbol{d}$ value is affected more than the $\boldsymbol{t}$ value. In this study, the amount of translation along the FHA $\boldsymbol{t}$ over 2 months was 0.37 mm with 0.5 N and 0.29 mm with 1 N. Again, these values were not significantly different. The mean value of the shortest distance $\boldsymbol{d}$ from the FHA to the tooth coordinate origin calculated weekly for 2 months was 14.63 mm with 0.5 N and 14.74 mm with 1 N. These values were not significantly different.

The direction vector of the FHA could indicate the axis of rotation that was most affected in 3D transformation (Hayashi *et al.*, 2004b, 2005). A component value of +1 or −1 indicates that the FHA is parallel to the corresponding coordinate axis, a value of 0 means that the FHA is perpendicular to the coordinate axis, and a value of +0.707 or –0.707 means that the FHA is at 45 degrees to the coordinate axis. In this study, signed rather than absolute component values of the direction vector were used to evaluate FHA behaviour. When the value of the v_x component is equal to −1, the FHA is parallel to the *x*-axis. This means that only tipping movement occurred. There were no significant differences between the two forces about v_z. However, there was a significant difference in the v_x and v_y components of the direction vector of the FHA between the two forces. The average value of the v_x component of the FHA direction vector calculated for all subjects from baseline to 2 months of canine movement was −0.25 with 0.5 N and −0.57 with 1 N ($P < 0.05$), indicating that the canine was likely to incline distally during tooth retraction with a force of 1 N compared with a force of 0.5 N. The average value of the v_y component of the FHA direction vector calculated for all subjects from baseline to 2 months of canine movement was 0.61 with 0.5 N and 0.37 with 1 N ($P < 0.05$). This result indicated that the canine was likely to incline palatally during retraction with a force of 0.5 N compared with a force of 1 N. Interestingly, this aspect of tooth movement was not detected by the *XYZ* system.

Discussion

To estimate the effective force for orthodontic tooth movement, it is important to consider not only the magnitude or type of force but also the type of mechanics used. There are two main types of canine retraction mechanics, friction (sliding) and frictionless (loop) mechanics; sliding mechanics were used in this investigation. Although several studies have demonstrated a significant decrease in friction with self-ligating brackets (Shivapuja and Berger, 1994; Pizzoni *et al.*, 1998; Thorstenson and Kusy, 2001), brackets with a conventional design were used. Previous investigations have suggested that the canine is retracted faster with loop mechanics than with sliding mechanics (Ziegler and Ingervall, 1989; Rhee *et al.*, 2001). However, these results were obtained with relatively large forces. Generally, retraction forces greater than 3 N result in a lag phase caused by necrotic tissue at the periodontal ligament (Gianelly, 1969). Additionally, a light continuous force can be easily applied with sliding mechanics using a NiTi closing coil spring. Furthermore, sliding mechanics offer the advantage of controlling canine rotation (Rhee *et al.*, 2001; Hayashi *et al.*, 2004*a*).

Although the optimal force for orthodontic tooth movement may differ for each tooth and for each individual patient, continuous low-magnitude force has been shown to be effective for tooth movement without undesirable side effects (Iwasaki *et al.*, 2000). Canine retraction is a common orthodontic procedure. If canine retraction is carried out with no simultaneous tipping or rotation, not only is the subsequent treatment plan facilitated but also the treatment

Table 2 Finite helical axis (FHA) system parameters: the amount of distal movement (mm), rotation around the FHA (degrees), translation along the FHA (mm), the shortest distance from the space coordinate system to the FHA (mm), and the components of the direction vector of the FHA over a 2-month treatment period.

FHA system parameters	0.5 N		1 N		P	
	Mean	SD	Mean	SD		
Distal movement of canine crown tip (mm)	3.17	0.60	3.38	0.71	0.199	NS
Rotation around the FHA $\boldsymbol{\theta}$ (degrees)	12.24	2.14	12.91	2.30	0.478	NS
Translation along the FHA $\boldsymbol{t}$ (mm)	0.37	0.17	0.29	0.13	0.223	NS
Shortest distance from the space coordinate origin to the FHA $\boldsymbol{d}$ (mm)	14.63	7.54	14.74	8.01	0.877	NS
Direction vector of the FHA v_x	−0.25	0.22	−0.57	0.20	0.048	*
Direction vector of the FHA v_y	0.61	0.21	0.37	0.22	0.042	*
Direction vector of the FHA v_z	0.61	0.29	0.58	0.30	0.234	NS

*$P < 0.05$. NS, not significant.

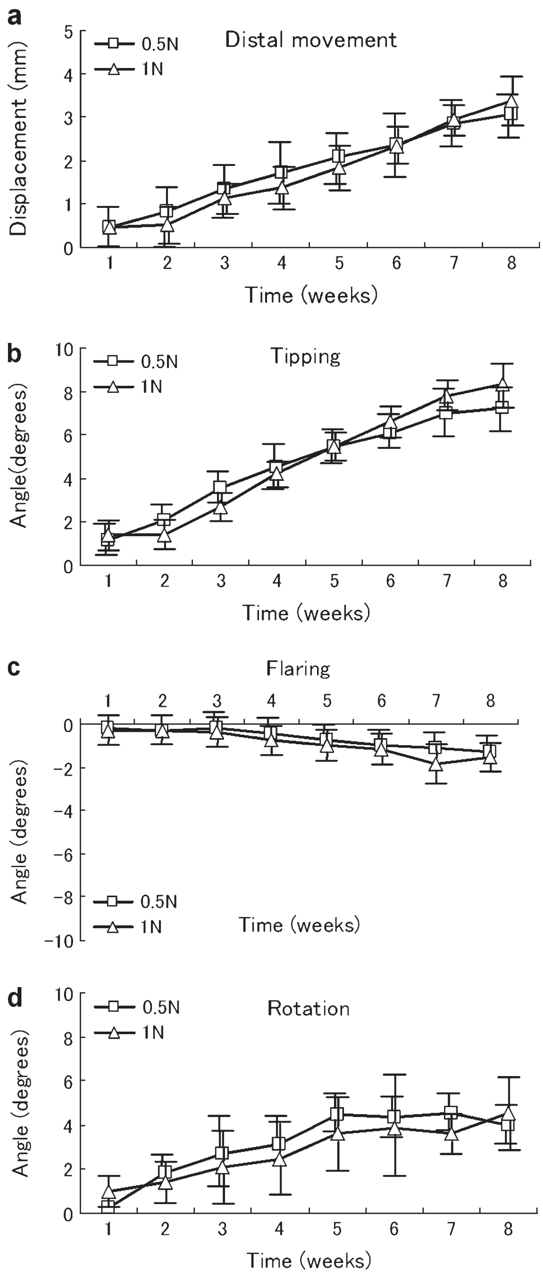

Figure 4 Average canine retraction by week (*XYZ* system) with force system of 0.5 and 1 N (error bars indicate the population standard deviation from the averages). (a) distal movement, (b) tipping, (c) flaring, and (d) rotation.

period can be shortened. Iwasaki *et al.* (2000) found that effective canine retraction could be produced with low continuous forces. Interestingly, in that study, force magnitudes of 0.18 and 0.6 N were found to produce effective tooth movement. A continuous force of 0.6 N was especially effective for canine retraction, and produced average distal tooth movement velocities of 1.27 mm/month with minimum simultaneous tipping or rotation. However, further research is required to support these results since the analysis of tooth movement in that report was based on the *XYZ* system and an unstable measuring reference.

During canine retraction, undesired tipping or rotation was minimized with both force magnitudes (Table 1). However, the canine retracted with a force of 1 N tended to tip distally compared with that treated with 0.5 N. While the tipping movement with 1 N (8.22 degrees) was adequately controlled, a smaller force can control undesirable side effects since the use of a smaller force may eliminate deflection of the main archwire. However, these results were obtained using the *XYZ* system, which means that the findings were influenced by the rotation sequence of the coordinate axis, as described previously. To establish the reliability of these results, it may be useful to compare the *XYZ* and FHA systems. The description of 3D motion using the FHA is independent of the chosen coordinate system since the values of the helical axis and the translation along the axis do not change with coordinate transformations. Thus, the FHA parameters are the same for all points on a moving body at any particular time point. This invariance makes it possible to compare tooth movement among different individuals.

Canine distal movement calculated using the FHA parameters was almost the same as that found using the *XYZ* system for both force magnitudes. This result demonstrates that the FHA parameters in this study were mathematically accurate and a valid representation of canine movement since movement of the origin was not influenced by computing rotation based on the *XYZ* system, i.e. the 3D distance between two arbitrary points is constant regardless of the coordinate system (description system) used. However, it is well-known that the rotational parameters and direction cosines of the FHA are sensitive to measurement errors. For each of these parameters, De Lange *et al.* (1990) derived analytical relationships for their standard deviations. To estimate the measurement error in this study, these standard deviations were calculated at each movement step. Large errors can occur in calculations of FHA direction vectors for small rotations about the FHA (less than 1 degree). Even for small rotation points (within 5 per cent of all points in this study), the maximum error of the FHA direction vector was only ±0.22 degrees.

With the FHA system, there were significant differences in the mean values of the components of the direction vector of the FHA (v_x and v_y) between the force magnitudes of 0.5 and 1 N (Table 2). The result regarding v_x means that the canine retracted with a force of 1 N clearly tended to tip distally compared with a force of 0.5 N. This supports the result with the *XYZ* system. The findings regarding v_y mean that the canine retracted with a force of 0.5 N tended to tip palatally compared with a force of 1 N. The primary reason for this may be the difference in friction between these force magnitudes (Pratten *et al.*, 1990). In sliding mechanics, friction between the canine bracket and the archwire may lead to distal sliding of the main archwire. In fact, the incisors were retracted distally during canine retraction in all subjects. Distal sliding of the main archwire produces a force on the canine in a palatal direction since the anterior section of the main archwire is narrower than the posterior section.

However, it is necessary to consider the limitation of this relatively small study. An investigation with a larger sample size might be required to support the findings.

Conclusion

While the *XYZ* and FHA systems each give a comprehensive 3D description of rigid body movement, the FHA system can provide more detailed information on tooth movement. While the *XYZ* system may be more intuitive for the clinician, the FHA system is biomechanically correct. In this study, the combination of these two different approaches for describing tooth movement clearly showed a difference between a light continuous force of 0.5 and 1 N. While both force magnitudes were effective for canine retraction in sliding mechanics, the results suggest that a force magnitude of 0.5 N was more effective for eliminating undesirable side-effects (excessive tipping and lag phase) during canine retraction.

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