

A new method for analyzing change in dental arch form

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Our method was developed and subsequently computerized in a system incorporating graphics and quantitative indexes to analyze change in dental arch form as a result of orthodontic treatment and subsequent relapse. The cubic spline function was used to model the form of the dental arches. On the basis of digitized data, splines were fitted to the dental arches before treatment, immediately after treatment, and at least 2 years after removal of retention. The fit of each spline was evaluated through the use of a set of normals to the curve and their average length. The change in form between the registrations was computed, and superimposed graphics of the three records were produced to visually indicate the magnitude of the changes in addition to numeric indexes that reflected the changes. Interjudge and intrajudge reliability were examined and found to be within acceptable limits. (*Am J Orthod Dentofacial Orthop* 1998;113:394-401.)

Many types of qualitative descriptions, geometric constructions, and mathematical curves have been proposed to describe the form of the normal dental arch. One of the more recently used methods for mathematical representation of arch form is the cubic spline curve. This curve consists of separate cubic polynomial segments connecting a series of points called knots in such a manner that the curve is smooth through the knots. A knot is a point through which the curve is forced to pass, so that different sets of knot points generate different curve configurations.

Many of the older methods used for representation of dental arch forms used mathematically symmetric functions. Many authors¹⁻⁵ have used a parabola of the form $x^2 = -2py$ to describe the shape of the dental arches. Others⁶⁻¹⁴ have used the catenary of the form $y = (e^x + e^{-x})/2$ as a means of describing arch form, and the use of an ellipse of the form $(x^2/a^2) + (y^2/b^2) = 1$ has also been suggested.^{2,4,5,12,13} Brader¹⁵ proposed the use of the trifocal ellipse. More recently, De La Cruz et al.¹⁶ fit conic sections to dental arches and evaluated the scale and eccentricity parameters. Common among these methods was the mathematical symmetry

present in the geometric figures obtained as a representation of arch form.

It has been shown that symmetry of the dental arches may not be a tenable assumption. Asymmetry of the arches has been described by Hechter³ and Lavelle and Plant.¹⁷ Although the results of these studies suggest that symmetry of the dental arches is uncommon, a variety of mathematically symmetric curves has nevertheless been superimposed on the arches in an attempt to describe arch form. Because the cubic spline function does not necessarily produce a symmetric curve, it appears to be an ideal means for representation of dental arch form as it may more adequately reflect the actual shape of the arch. BeGole¹⁸ found that the cubic spline curve fit the arch forms of well-aligned dental arches with minimal error and that asymmetry of the arch had no effect on accuracy of fit. Even for malaligned arches, with judicious selection of knot points, Bobkin¹⁹ found that a cubic spline curve could be constructed that gave a reasonably accurate representation of the form of the arch. Coombs and Deming²⁰ also used cubic spline curves to represent dental arches and later to compute changes in arch form.

The posttreatment change in the orthodontically altered dentition has been extensively reported in the literature. Investigators are in general agreement that the form of the arch, like many other aspects of the dentition, demonstrates a propensity to return toward its pretreatment state after orthodontic movement. Traditionally, change in arch form has been analyzed in terms of the behavior of various linear dimensions such as arch width, depth,

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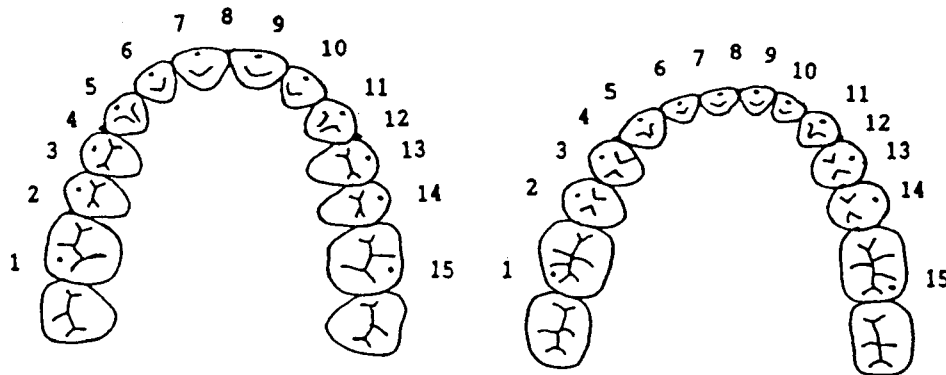


Fig. 1. Drawing intended to represent points digitized on occlusal photograph.

and circumference. However, many authors have cautioned that analyses of this type are incomplete and limited in description. Lavelle et al.²¹ stressed that single variate analysis involving the use of intertooth dimensions tended to treat the dental arch as a collection of discrete entities rather than a biological unit, not to mention the possibility of cross correlation between the variates. Lavelle²² and later Moyers and Bookstein²³ and Baluta and Lavelle²⁴ all maintained that any analysis of change in form must consider both size and shape changes. Sampson⁵ declared that changes in intertooth dimensions fail to distinguish size and shape changes and provide only a fraction of the information embodied in the changing curvature of an arch form.

In the development of a more complete evaluation of change in arch form, attention focused on geometric curves and more sophisticated analyses but they were not without significant limitations. Shapiro¹² used catenary curves and evaluated the associated y-intercepts but later²⁵ reported that his method was unsuccessful. Bobkin¹⁹ used cubic spline curves superimposed on the midpalatal raphe and registered on the incisal papilla; however, apart from comparing the slopes of the curves at the molar regions, the analysis focused primarily on evaluation of the fit of the splines. Cubic spline curves were also used by Coombs and Deming²⁰ who examined change in arch form with the use of multivariate statistical analyses of the coefficients in the equations of the spline curves. However, the mathematical complexity of such an approach makes it difficult to relate numeric results to any conceptualization of changes in form. This restriction applies as well to the multivariate statistical methods of Lavelle et al.²¹

Arch form and changes in this form reflecting both size and shape changes are of interest to orthodontic treatment planning and prognosis.

Other more complex and advanced methods have been used that reflect both size and shape changes and provide a more complete description of change in form than the traditional measures. These methods include Euclidean distance matrix analysis of Lele,²⁶ Lele and Richtsmeier,²⁷ Ferrario et al.²⁸⁻³⁰; finite element analysis used by Skalak et al.,³¹ Chevruud et al.,³² Moss et al.,³³ and Baluta and Lavelle²⁴; and Procrustes analysis of Siegel and Benson,³⁴ Chapman,³⁵ and Goodall.³⁶

Accordingly, the purpose of the present method was to use the cubic spline curve, which is independent of dental arch asymmetry, as the basis for a simple mathematical model to evaluate both treatment and posttreatment changes in the dental arches. The method is designed to be a compromise between the use of a few specific intertooth measurements and a wholly continuous evaluation of the changing curvature of an arch form. Because it is based on the division of the curve into many small segments represented by equally spaced points, the method approaches a continuous evaluation of arch form while retaining the capability for simple linear analysis. The method is not concerned with the change in position of any specific tooth or group of teeth or arch segment. Rather, the change in the entire configuration of the arch form is evaluated, and numeric quantities are attached to this change. In addition, the method allows for determination of relapse movement and a quantitative evaluation of the change in the system of superimposed arches.

METHODS

Data for the mathematical model arise from orthodontic records that consist of pretreatment (A) plaster casts; posttreatment (B) casts taken at the completion of fixed appliance orthodontic therapy; and postretention

Table I. Intrarater reliability with paired *t* tests

<i>Variable</i>	<i>Mean difference</i>	<i>SD</i>	<i>t</i>
Extraction (n = 12)			
Average normal A	-0.05	0.12	-1.52
Average normal B	0.03	0.05	2.22*
Average normal D	0.01	0.05	0.84
Arc length A	-0.02	0.75	-0.11
Arc length B	0.52	1.68	1.06
Arc length D	-0.23	1.03	-0.76
Nonextraction (n = 8)			
Average normal A	0.00	0.07	0.00
Average normal B	-0.01	0.03	-0.41
Average normal D	0.03	0.05	1.62
Arc length A	-1.06	1.95	-1.54
Arc length B	-0.25	0.46	-1.55
Arc length D	-0.64	1.15	-1.57

p* < 0.05.Table II.** Interrater reliability with paired *t* tests

<i>Variable</i>	<i>Mean difference</i>	<i>S.D.</i>	<i>t</i>
Extraction (n = 12)			
Average normal A	0.01	0.14	0.21
Average normal B	0.05	0.04	4.22*
Average normal D	0.04	0.08	1.74
Arc length A	0.03	1.02	0.09
Arc length B	1.05	2.85	1.28
Arc length D	1.05	3.51	1.03
Nonextraction (n = 8)			
Average normal A	0.04	0.08	1.41
Average normal B	0.06	0.15	1.11
Average normal D	0.03	0.09	1.12
Arc length A	-0.32	1.47	-0.61
Arc length B	-0.16	0.67	-0.69
Arc length D	0.04	0.34	0.34

**p* < 0.05.

(D) casts taken a minimum of 2 years after discontinuation of retention. Records for 65 subjects each having A, B, and D models for a total of 390 casts were used to develop the method. Models for each subject were photocopied with the occlusal plane resting on the glass and with a millimeter ruler placed in the field that is used to correct for magnification errors related to the copying process. The cast is placed in the center of the camera lens of the copying machine to minimize distortion in the copied image of the cast. From the photocopies, the following points (Fig. 1) are digitized: the incisal edge midpoints (points 6, 7, 9, and 10); the buccal cusp tips of the canines (points 5 and 11); the buccal cusp tips of the premolars (points 2, 3, 13, 14); and the distobuccal cusp tips of the first molars (points 1 and 15). These points had been marked on the dental casts before digitization to ensure proper location of the landmarks on the photocopies. Three extra points, known as the floating points 4, 8, and 12, add to the flexibility of curve construction and

are also digitized. These points are also represented in Fig. 1. Point 8 is usually the midpoint between the mesioincisal corners of the central incisors. However, if these teeth are judged to be excessively inclined relative to the lateral incisors, the point is located more over basal bone in a somewhat less extreme position to prevent significant aberration in the curvature of the generated arch form. Point 4 and contralaterally point 12 are usually placed in the embrasure between the cuspid and adjacent premolar on a line connecting the marks on these teeth, as long as the premolar-cuspid-lateral incisor alignment is judged to be acceptable. However, for a severely malaligned pretreatment arch with a lateral incisor sharply inclined toward the lingual, the respective point is located between the cuspid and lateral incisor in order to guide the curve around the cuspid toward the lateral incisor and thus keep the anterior form close to the outline of the arch. One of the primary advantages of the mathematical model is that it allows for subjective judgment on the part

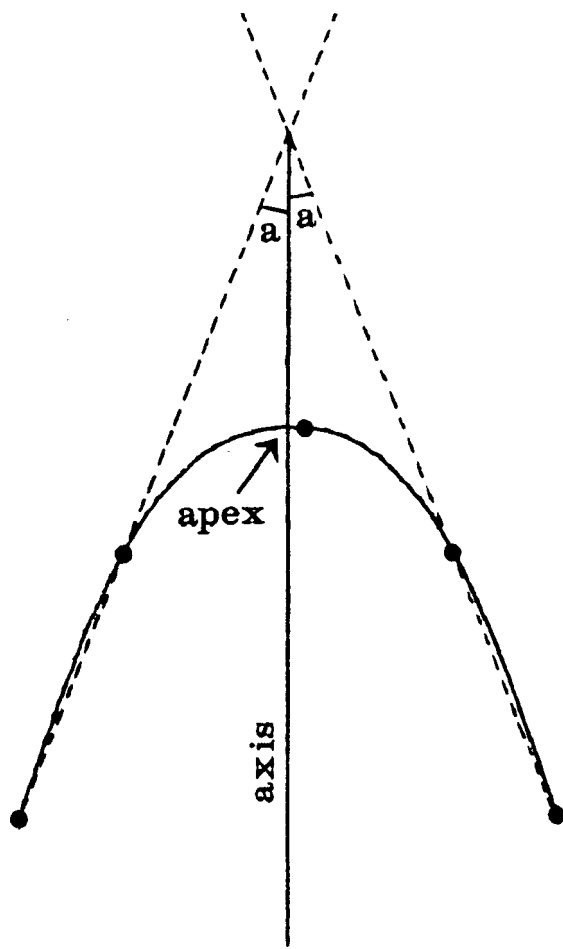


Fig. 2. Construction of superimposition references.

of the user. This judgment is reflected both in choice of knot points and in location of the three floating points, all of which the user may manipulate until the best fit of the curve is obtained. Best fit is objectively determined with a set of normals to the curve as described here.

A computer-based system was written to perform the analysis of change in arch form. A set of 15 points was previously marked on the dental casts to ensure proper location of landmarks on the photocopy. The 15 points on the photocopy are digitized interactively, and the user indicates which of the data points are to be used as knot points, making sure that points 1 and 15 are always included so as to complete the distal extensions of the spline curve. On specification of the knot points, the program interpolates a cubic spline over the knots. Two indexes are then computed to aid in the determination of the accuracy of the fit of the spline to the dental arch form. These are the sum of the length of normals to the curve from data points not used as knot points, and the average length of these normals. A normal is defined as a perpendicular to a tangent constructed on the curve so as

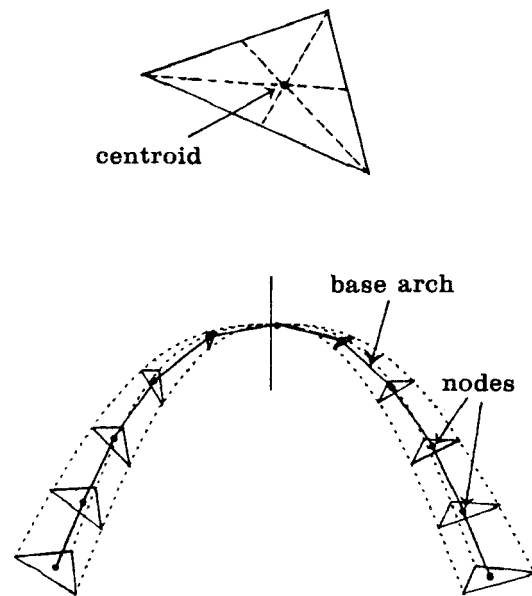


Fig. 3. Construction of base arch.

to minimize the distance from the exterior point to the curve. Low values for each index suggest a spline that portrays an accurate representation of the form of the arch. The assumption is that if a spline has a perfect fit, it passes precisely through all the data points.

The spline curves are then graphed by the computer program, and each plot is positioned over the photocopy of the digitized cast in order to visually assess approximation to the dental arch form. If it is thought that the curve does not sufficiently approximate the arch form or if the accuracy indexes are too large, a new set of knot points may be chosen by the user to generate a spline of different configuration. Fine-tuning adjustments to the curve by slight alterations in the positions of the three floating points if any were included as knots. In general, a smooth curve with an average normal length of 0.5 mm or less was the goal of B and D spline curves as suggested by BeGole.¹⁸ Because of varying degrees of dental malalignment present in the pretreatment records, a similar accuracy standard cannot be applied to the A spline. However, the objective remains to minimize the accuracy indexes and produce a smooth curve.

To show changes in arch form, the representative A, B, and D spline curves are superimposed. Many authors such as Shapiro,¹² Bobkin,¹⁹ and Lebret³⁷ oriented on the midpalatal raphe of the maxillary arch and registered on some presumed stable landmark such as the distal of the incisive papilla. However, no analogous system is available for use in the mandibular arch. Bobkin¹⁹ and Hechter³⁸ mechanically transferred the maxillary superimposition references to the mandible, but occlusal changes or differential jaw growth that affect the relative position of the mandible could produce misleading information about

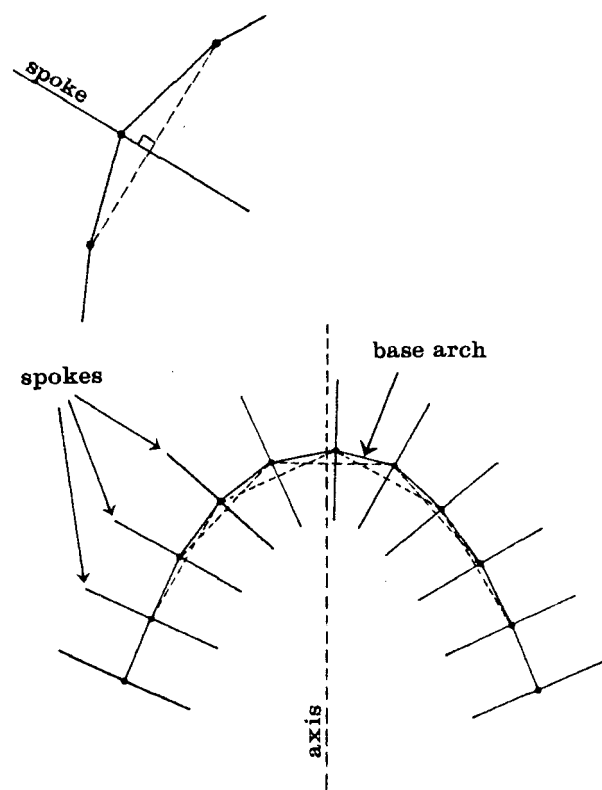


Fig. 4. Construction of spokes through nodes of base arch.

changes in lower arch form. To circumvent these problems, some authors such as Brader,¹⁵ Speck,³⁹ and Davis⁴⁰ used superimposition references constructed from only data points on the dental arch so that both maxillary and mandibular analyses could be performed. For this reason, the analysis presented herein does likewise. After superimposition of the three spline curves representing the three sequential arches, the computer program carried out the analysis of change in arch form.

In order to establish reliability of the model for measurement, a subset of 10 models including both maxillary and mandibular arches was digitized twice by a second examiner. The sample included 6 extraction cases (12 arches) and 4 nonextraction cases (8 arches). Two variables were assessed: the average normal on which error of fit of the model is based, and the arc length at each of three stages, A, B, and D. Results of the second examiner are presented in Table I and show the mean difference between the two sets of measurements, the standard deviation of these differences, and the paired *t* test value, which tests if the difference between the two measurements is zero. In only one instance was a statistically significant difference found, and that was for the average normal at stage B for extraction cases. The average magnitude of the difference between the two sets

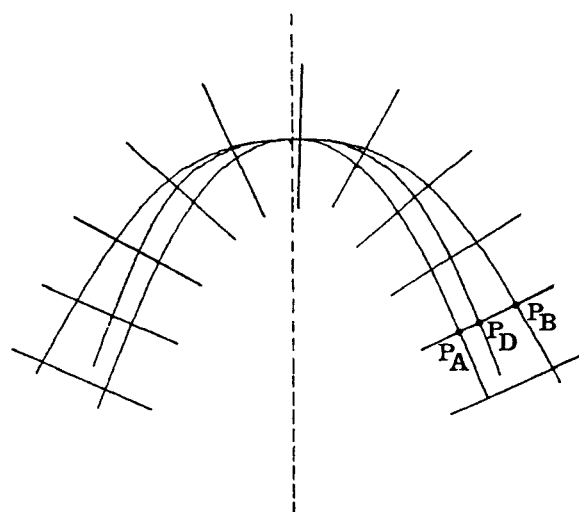


Fig. 5. System of radial spokes intersecting superimposed spline curves.

of measurements was 0.03 mm, which is of little consequence. The mean of this examiner's two sets of measurements was then compared with that of the first examiner, and results of interrater reliability using paired *t* tests are shown in Table II. In only one instance was the mean difference between examiners significant, and that was also for the average normal at stage B for extraction cases. The average difference between examiners for this variable was also of very low magnitude, 0.05 mm. It therefore appears that the curves are well identified at all three stages both within and between investigators. Extraction casts were perhaps less reproducible than nonextraction casts as the only significant differences were found for extraction cases. Pretreatment casts appear to be as reproducible as subsequent casts.

THEORETICAL DEVELOPMENT OF THE MODEL

The Cubic Spline

The cubic spline is described as a mathematical model of the physical device used by draftspersons to draw smooth curves. The physical spline consists of a thin flexible strip of wood or plastic that is laid over graph paper and held in such a way that it is forced to pass through certain data points. The draftsperson then traces along the spline to obtain the curve. Data for the mathematical model correspond to the data points through which the physical spline of the draftsman is forced to pass. These points are called the knots of the spline.

In order to mathematically fit a spline to a set of data points, a set of knots must be selected based on the restriction that $x_1 < x_2 < \dots < x_n$. Once chosen,

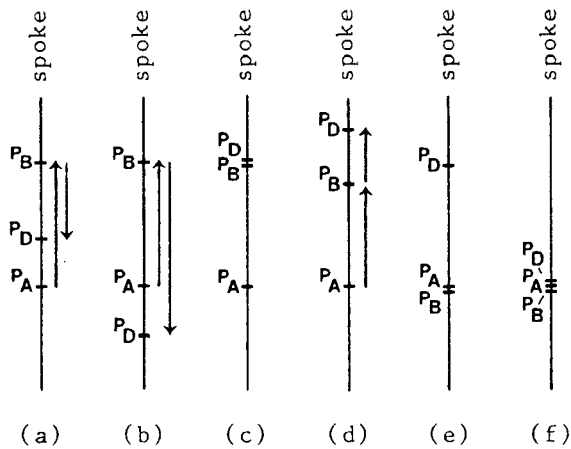


Fig. 6. Various arrangements of intersection points of three splines on any given spoke.

a cubic polynomial is defined between each successive pair of knots, and subsequently the curve is smoothed at the knot resulting in a cubic spline that passes precisely through the values chosen as knots. Thus, the fit of a spline resides, partially at least, in the choice of knot points in addition to their number. For a complete description of the mathematics underlying the cubic spline, see BeGole,⁴¹ Handscomb,⁴² Shampine and Allen,⁴³ and deBoor.⁴⁴

Superimposition

After generation of suitable A-, B-, and D-spline curves, the program superimposes the curves over a common orientation line and registration point (Fig. 2). The orientation line is constructed as follows. On each side of the arch, a line is drawn connecting the terminal (molar) knot point with the knot point closest to the cuspid data point on that side; if the cuspid point serves as a knot point, then it is connected to the molar knot point. The two lines so formed are extended to intersect above the curve, and the bisector of the interior angle forms the orientation line and constructed axis of the curve. The intersection of the constructed axis with the curve forms the apex and the registration point for superimposition. In the computer algorithm, superimposition is simply a matter of transforming the digitized data points for each of the A-, B-, and D-arches to a new coordinate system, so that the constructed axis forms the Y-axis and the apex becomes the origin of the system. The transformation is achieved by rotating the old axes X and Y through the angle θ using an orthogonal transfor-

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CASE 1131-A/U
KNOTS : 1 4 6 8 10 13 15
SUM/NML - 11.01 MM
AVG/NML - 1.57 MM
    
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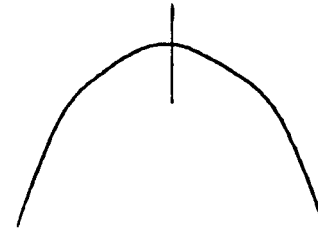


Fig. 7. Computer-generated curve from pretreatment record shows knots and accuracy indexes. Seven knot points were used, and sum of normals and average normal distance are printed.

mation matrix so that the relationship between the old (X,Y) and new (X',Y') values may be expressed as:

$$\begin{aligned}
 X' &= X\cos\theta + Y\sin\theta \\
 Y' &= -X\sin\theta + Y\cos\theta
 \end{aligned}$$

The Base Arch

The purpose of the base arch is to enable the construction of a set of spokes, the measurement of which produces various indexes reflecting change in arch form. The base arch is constructed as follows. On superimposition of A-, B-, and D-spline curves, the program performs the analysis of change in arch form as follows. Each spline curve is divided with 100 equally spaced representation points, and corresponding points between the curves are connected with straight lines to form 100 triangles around the perimeter of the curves (Fig. 3). The centroids of the triangles are then connected in series to form a base arch consisting of 100 nodes and 99 straight-line segments. The centroid of a triangle is defined as the common point of intersection of the lines connecting each apex with the midpoint of the opposite side. In the event that three connected representation points form a straight line instead of a triangle, the midpoint of the line is used as the node of the base arch. Finally, through each internal node of the base arch a spoke is drawn perpendicular to the line connecting adjacent nodes on either side (Fig. 4); for the terminal nodes, the spoke is drawn perpendicular to the respective terminal straight-line segment of the base arch. Thus there are 100 radially directed

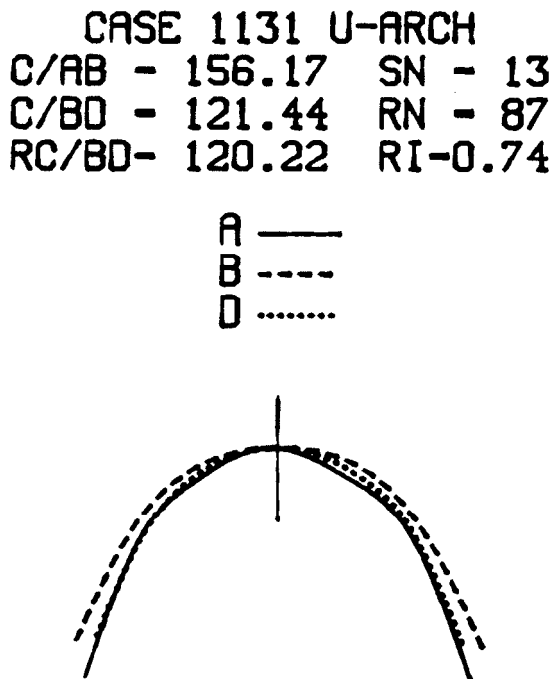


Fig. 8. Superimposition of spline curves and indexes of change in arch form. (A, pretreatment; B, posttreatment; D, postretention)

spokes constructed around the superimposed spline curves (Fig. 5); this arrangement forms the basis for the analysis of arch form change.

Treatment Change, Posttreatment Change

If a spoke intersects the A-curve at point P_A , the B-curve at point P_B , and the D-curve at point P_D (Fig. 5), the sum of lengths P_A - P_B for all 100 spokes around the curves is taken to represent the total treatment change (C_{AB}) in arch form from the A-curve to the B-curve. Similarly, the sum of lengths P_B - P_D represents the total posttreatment change (C_{BD}) from the B-curve to the D-curve. In the computer program, two intersection points on a spoke must be at least 0.35 mm apart in order to be included in the sums. The reason for this is that the line thickness on the plots of the superimposed curves is approximately 0.2 mm. To ensure unambiguous separation between the drawn curves intersecting a spoke, a critical minimum distance of 0.15 mm between curves, analogous to 0.35 mm between points, was empirically chosen. This results in a more conservative figure for arch form change. Two curves were considered to be coincident on a spoke if the intersection points were less than 0.35 mm apart.

Rebound Number and Stability Number

Further useful information is obtained by examining the relative positions of P_A , P_B , and P_D on each spoke. If P_D is located between P_A and P_B (Fig. 6, A) or P_D is counterpositioned to P_B (Fig. 6, B), then it is inferred that the part of the D-curve on that spoke had shifted in the opposite direction of the treatment change from the A-curve to the B-curve. The number of such spokes with either arrangement of P_A , P_B , and P_D is termed the rebound number (RN), representing how much of the D-curve exhibits a relapse shift back toward the A-curve. In the same manner, the stability number (SN) is figured by counting the number of spokes with P_B and P_D coincident (Fig. 6, C), and it represents how much of the D-curve remains unchanged with respect to the B-curve. It should be noted that if points P_A and P_B alone are coincident on a spoke (Fig. 6, E), or if P_D is located on the opposite side of P_B away from P_A on a spoke (Fig. 6, D), no relationship is assigned to either spoke. Moreover, for simplicity of computation, if points P_A , P_B , and P_D are all coincident on a spoke (Fig. 6, F), then that spoke is counted in the SN.

Rebound change

The identification of spokes with PD exhibiting the relapse shift (Fig. 6, A and B) allows another quantity to be computed. The rebound change (RC_{BD}) is figured by summing the lengths P_B - P_D on these spokes, and it represents the relapse component of the posttreatment change C_{BD} . For ease of interpretation of the relapse index (RI) computed below, the length P_B - P_D can be less than or equal to, but never greater than, the length P_A - P_B on a spoke used to compute RC_{BD} .

Relapse index

The RI represents the overall relapse change in the system of superimposed spline curves and is calculated according to the empirically derived formula

$$RI = (RC_{BD}/C_{AB})(RN/[100-NC])$$

The first component of the formula expresses the RC_{BD} as a proportion of the original C_{AB} . The second component expresses the RN as a proportion of the total number of spokes (100) less the number of spokes NC with coincident intersection points P_A , P_B and P_D (Fig. 6, F). Careful inspection will show that each component in the formula will vary between 0 and 1, and hence RI itself will

compute to a value between 0 and 1 inclusive. An index of 1 would indicate total relapse of the D curve back to the A curve or even beyond. An index of 0 would indicate no relapse changes in the D curve whatsoever. Most cases would be expected to have a relapse index between 0 and 1, showing some amount of relapse of the D curve.

The Graphics

Fig. 7 shows the representation of a single spline curve with the associated indexes of accuracy, whereas Fig. 8 shows the superimposition of three splines that demonstrates change in the form of the dental arches. Indexes of change are printed on the superimposed graphics.

CONCLUSION

Although the mathematical function with the cubic spline has been found to adequately model the form of the dental arches and is the basis for the new method of analysis, further studies are required in order to establish the reliability of the new method. In addition, the ability of the method to distinguish between men and women, maxillary and mandibular arches, and extraction and nonextraction cases will be evaluated. It is important also to relate the results of the new method to traditional linear measurements used to describe change in the form of the arches.

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