Assessment of second-order clearances between orthodontic archwires and bracket slots via the critical contact angle for binding

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Abstract: Twenty-six archwires and 24 brackets were selected from among the hundreds of products available that nominally have from 18 to 22 mil bracket slots and 14, 16, 17, 18, 19, and/or 21 mil archwire sizes. After the archwires and brackets were dimensioned, a minimization-maximization algorithm was applied to the measurements in order to establish the likely boundaries of the critical contact angle for binding (θ_c) as defined by the presence and absence of second-order clearance. From among the myriad archwire-bracket permutations possible, 64 combinations were identified—20 using the bracket slot as the controlling dimension and 44 using the bracket width. Using a previously derived mathematical expression that relates the dimensions of each archwire-bracket couple to its calculated θ_c, the corresponding sets of indices were plotted. The results show that the maximum value of the calculated θ_c can never exceed about 5°, or else sliding mechanics will always be hampered. Other outcomes were validated experimentally using 5 of the 64 archwire-bracket couples by measuring the resistance to sliding (RS) at 15 different contact angles (θ) ranging from $\theta=0°$ to $\theta=12°$ and by subsequently determining a measured θ_c. These values agreed with the calculated θ_c values. When the practitioner knows the θ_c, treatment time might be reduced because the teeth do not need to be over-aligned prior to employing sliding mechanics (i.e., by not making $\theta<<\theta_c$) and, further, because the contact angle beyond which the binding phenomenon retards or halts tooth movement does not need to be exceeded (i.e., by not making $\theta>\theta_c$) These results underscore the importance of exact wire and bracket dimensions on packaging; otherwise, sliding mechanics can be compromised by miscalculating θ_c.

Key Words: Archwires, Brackets, Clearances, Contact angles, Sliding mechanics

Within the past 30 years, several investigators have shown that resistance to sliding increases substantially as the contact angle (θ) between an archwire and a bracket slot increases.[1-6] From such results, Frank and Nikolai[3] and others[4,5,7-9] showed that as θ increased, resistance to sliding was greater with stainless steel than with titanium alloy wires. Although other experimental observations have been documented, no rigorous mathematical approach or scientific principle has been forthcoming that relates the specific dimensions of archwire-bracket couples and their angulations to sliding performance.

In 1998, a theory was reported that was based on the relative geometry of the archwire-bracket couple.[10] This theory showed that the θ at which opposing bracket tie-wings engaged the archwire and the second-order clearance reduced to nil could be described by just three geometric parameters (Figure 1): the dimension of the archwire that engages the floor of the slot ("SIZE"), the corresponding bracket dimension at the floor of the slot ("SLOT"), and the mesiodistal width of the bracket ("WIDTH"). The closed-form solution for this critical contact angle for binding (θ_c) could be expressed as

$$\theta_c = \cos^{-1} \frac{(\mathrm{SIZE})^2 - (\mathrm{WIDTH})^2}{(\mathrm{SIZE})\,(\mathrm{SLOT}) \pm (Z)^{0.5}} \quad (1)$$

in which

$$Z = (\mathrm{WIDTH})^2\,[-(\mathrm{SIZE})^2 + (\mathrm{SLOT})^2 + (\mathrm{WIDTH})^2]. \quad (2)$$

A mathematically simpler, but open-form solution of eqns. (1) and (2) was subsequently derived as

$$\frac{\mathrm{SIZE}}{\mathrm{SLOT}} = \frac{\mathrm{WIDTH}}{\mathrm{SLOT}} (\sin\theta_c) + \cos\theta_c. \quad (3)$$

(In the closed-form solution, θ_c can be obtained directly by substituting the values of the three geometric parameters into eqns. 1 and 2. In contrast, in the open-form solution, only trial and error can be used to solve for θ_c in eqn. 3. Either solution is exact, however.)

From eqns. 1 and 2 or eqn. 3, θ_c was plotted as a function of two dimen-

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Submitted: November 1997; **Revised and accepted:** June 1998
Angle Orthod 1999;69(1):71-80.

Table 1
Archwires measured

Company (location)	Archwire name	Material*	Nominal SIZE (mils)	Actual SIZE (mils)	Tagged archwires
American Orthod. (Sheboygan, Wisc)	Gold Tone Round	SS	16	15.60 ± 0.08	c
	Standard Edgewise	SS	21 x 25	20.99 ± 0.02	
	Standard Round	SS	14	13.94 ± 0.02	
Dentaurum (Pforzheim, Germany)	Remanium	SS	16	15.77 ± 0.03	
	Remanium	SS	16 x 22	16.20 ± 0.03	d
	Remanium	SS	17 x 25	17.18 ± 0.08	f
	Remanium	SS	19 x 25	19.18 ± 0.11	j
	Remanium	SS	21 x 25	21.75 ± 0.00	l
GAC (Central Islip, NY)	Nubryte Gold	SS	14	14.10 ± 0.03	b
Ormco/Sybron (Glendora, Calif)	Round	SS	14	13.98 ± 0.03	
	Round	SS	16	16.20 ± 0.03	
	Round	SS	18	18.18 ± 0.03	h
RMO (Denver, Colo)	Elgiloy—Blue	Co-Cr	16 x 16	15.85 ± 0.11	
	Elgiloy—Blue	Co-Cr	21 x 25	20.84 ± 0.04	k
	Elgiloy—Green	Co-Cr	18	17.60 ± 0.04	g
	Elgiloy—Red	Co-Cr	14	13.79 ± 0.02	
	Elgiloy—Yellow	Co-Cr	16 x 22	15.99 ± 0.09	
	Elgiloy—Yellow	Co-Cr	17 x 25	16.73 ± 0.05	
	Elgiloy—Yellow	Co-Cr	18 x 25	17.88 ± 0.04	
	Elgiloy—Yellow	Co-Cr	19 x 25	18.48 ± 0.08	i
	Tru-chrome	SS	14	13.77 ± 0.03	a
	Tru-chrome	SS	17 x 25	16.23 ± 0.03	e
Unitek/3M (Monrovia, Calif)	Standard Rectangular	SS	17 x 25	16.90 ± 0.04	
	Standard Rectangular	SS	16 x 25	15.98 ± 0.15	
	Standard Rectangular	SS	19 x 25	18.99 ± 0.06	
	Standard Square	SS	16 x 16	16.11 ± 0.08	

* SS = stainless steel (iron, chromium, and nickel alloy)
Co-Cr = cobalt-chromium alloy

sionless parameters: the engagement index (SIZE/SLOT)—that is, how well the wire SIZE fits in the bracket SLOT; and the bracket index (WIDTH/SLOT)—that is, how much larger the mesiodistal bracket WIDTH dimension is than its coronal-apical SLOT dimension (Figure 2). For nominal archwire SIZES (14, 16, 17, 18, and 19 mils), nominal bracket SLOTS (18 and 22 mils), and assumed bracket WIDTHS (125 mils for 3s, 4s, and 5s, and 250 mils for 1s), the limits of θ_c were calculated and ranged from $\theta_c = 0°$ to $\theta_c = 4.5°$. This outcome indicated that if the practitioner wanted to use sliding mechanics without any binding, he or she always had to align and level teeth specifically within this aforementioned envelope. Moreover, if the practitioner exceeded the characteristic value of θ_c for a given wire-bracket couple, sliding would be increasingly compromised as θ increasingly exceeded θ_c.

Having established the mathematical relationship, the present effort seeks to determine the limits of θ_c values from actual archwire-bracket couples of 3s, 4s, and 5s. A surprising shift is observed because, for whatever reason, the actual SIZES and actual SLOTS are not always what vendors label them to be. Ultimately, experimental measurements from five wire-bracket couples confirmed that these measured θ_c values are in agreement with the calculated θ_c values that were obtained from the previously derived eqns. 1 - 3.

Materials and methods

Archwires and brackets

Twenty-six archwires were randomly selected from among six major vendors and two major alloy groups (Table 1). These selections included at least one wire from each of the nominal wire SIZES. The actual SIZES of the wires that engage any bracket were measured to the nearest 0.01 mil at six locations using a Sony μ-Mate micrometer (Sony Magnescale America, Inc, Orange, Calif).

Table 2
Brackets measured

Company (location)	Bracket name	Material*	Nominal SLOT (mils)	Actual SLOT (mils)	Actual WIDTH (mils)	Tagged brackets
"A"-Company	Starfire	SCA	18	18.2 ± 0.2	121.0 ± 0.0	
(San Diego, Calif)	Starfire	SCA	22	22.4 ± 0.2	121.0 ± 0.0	
American Orthod.	20/20	PCA	18	17.8 ± 0.2	129.0 ± 0.0	A (7.25)**
(Sheboygan, Wisc)	20/20	PCA	22	21.5 ± 0.1	129.0 ± 0.0	E (6.00)
Dentaurum	Fascination	PCA	18	20.7 ± 0.5	136.0 ± 0.0	
(Pforzheim, Germany)	Fascination	PCA	22	23.6 ± 0.5	137.0 ± 0.0	
	Rematitan (7°T***)	Ti	18	20.9 ± 0.3	142.5 ± 0.5	B (6.82)
	Rematitan (17°T)	Ti	18	20.1 ± 0.3	143.0 ± 0.0	D (7.11)
	Ultra-minitrim	SS	18	20.4 ± 0.2	105.0 ± 0.0	C (5.15)
	Ultra-minitrim	SS	18.5	19.5 ± 0.1	146.3 ± 0.6	
	Ultra-minitrim (7°T)	SS	18.5	18.6 ± 0.2	150.0 ± 0.0	
	Ultra-minitrim (17°T)	SS	18.5	19.9 ± 0.1	151.5 ± 0.0	
	Ultra-minitrim	SS	22	23.7 ± 0.2	146.0 ± 0.0	F (6.16)
GAC	Allure III	PCA	22	22.8 ± 0.1	148.5 ± 0.0	
(Central Islip, NY)						
Ormco/Sybron	Lumina Twin	PCA	18	18.3 ± 0.5	127.7 ± 0.3	
(Glendora, Calif)	Lumina Twin	PCA	22	22.3 ± 0.1	131.5 ± 0.0	
RMO	Quasar	PCA	18	18.1 ± 0.2	126.5 ± 0.0	
(Denver, Colo)	Quasar	PCA	22	21.9 ± 0.1	130.0 ± 0.0	
TP Orthod.	Advant-edge	PCA	22	22.8 ± 1.1	156.3 ± 0.3	H (6.86)
(LaPorte, Ind)						
Unitek/3M	Mini Uni. Twin	SS	22	23.2 ± 0.0	121.0 ± 0.0	G (5.22)
(Monrovia, Calif)	New Ceramic	PCA	18	18.0 ± 0.5	142.7 ± 0.6	
	Transcend 6000	PCA	18	18.3 ± 0.5	141.0 ± 0.0	
	Transcend 6000	PCA	22	22.6 ± 0.1	139.9 ± 0.0	
	Victory Mini Twin	SS	18	18.3 ± 0.2	139.0 ± 0.0	

* SCA = single crystal alumina, a.k.a. sapphire
PCA = polycrystalline alumina
Ti = commercially pure titanium
SS = stainless steel (iron, chromium, and nickel alloy)
** Bracket Index = WIDTH/SLOT, as determined by actual dimensions reported here.
*** T = pretorqued bracket

To complement these archwires, 24 brackets were selected from among eight major vendors and four major material groups (Table 2). Both standard bracket SLOTS were included as well as three 18.5 SLOTS. The actual SLOTS of the brackets were measured to the nearest 0.1 mil three times on each side for a total of six measurements per bracket, using the optics of a Kentron microhardness tester (Kent Cliff Labs, Peekskill, NY). The actual WIDTHS of these brackets were measured to the nearest 0.1 mil three times using Starrett calipers (LS Starrett Co, Athol, Mass).

Minimization-maximization algorithm

Each SIZE, SLOT, and WIDTH measurement was reported as a mean plus-or-minus its standard deviation in mils. Having made the actual measurements (Tables 1 and 2, columns labeled "Actual"), minima and maxima were identified that corresponded for each nominal bracket SLOT (18 or 22 mils) to the nominal archwire SIZES available (14, 16, 17, 18, 19 and/or 21 mils). These archwires and brackets are labeled "Tagged" in the right-hand margins of Tables 1 and 2 as lower case and upper case letters, respectively. Note that, with regard to archwires, six nominal wire SIZES and hence six minima (labeled a, c, e, g, i, and k) and six maxima (labeled b, d, f, h, j, and l) are possible. With regard to brackets, however, two minima (labeled A and E) and two maxima (labeled B and F) are possible for each bracket if the SLOT dimension controls; whereas two minima (labeled C and G) and two maxima (labeled D and H) are possible for each bracket if the WIDTH dimension controls. Thereby, the number of permutations is eight, unless a minima or maxima SLOT dimension coincidentally corresponds to a minima or maxima WIDTH dimension. In Table 2, the actual bracket indices (i.e., the WIDTH/SLOT ratios) are shown in parentheses alongside the tagged brackets.

Because the minima and maxima of actual SLOT and actual WIDTH do not coincide, separate analyses of SLOT values A, B, E, and F are required, while allowing the WIDTHS to assume their corresponding actual values. Similarly, separate analyses of WIDTH values C, D, G, and H are required, while allowing the SLOTS to assume their corresponding actual values. Discounting the eight combinations in which the wire SIZE will not engage the bracket SLOT leaves 64 viable combinations:

Using SLOT values as the controlling dimension—

- With the A brackets, b, d, and f wires
- With the B brackets, a, c, e, g, i, and k wires
- With the E brackets, b, d, f, h, and j wires
- With the F brackets, a, c, e, g, i, and k wires

Using WIDTH values as the controlling dimension—

- With the C brackets, a, b, c, d, e, f, g, h, i, and j wires
- With the D brackets, a, b, c, d, e, f, g, h, i, and j wires
- With the G brackets, a, b, c, d, e, f, g, h, i, j, k, and l wires
- With the H brackets, a, b, c, d, e, f, g, h, i, j, k, and l wires

The above combinations include all of the A, C, G, and H permutations that *also* represent the minima and

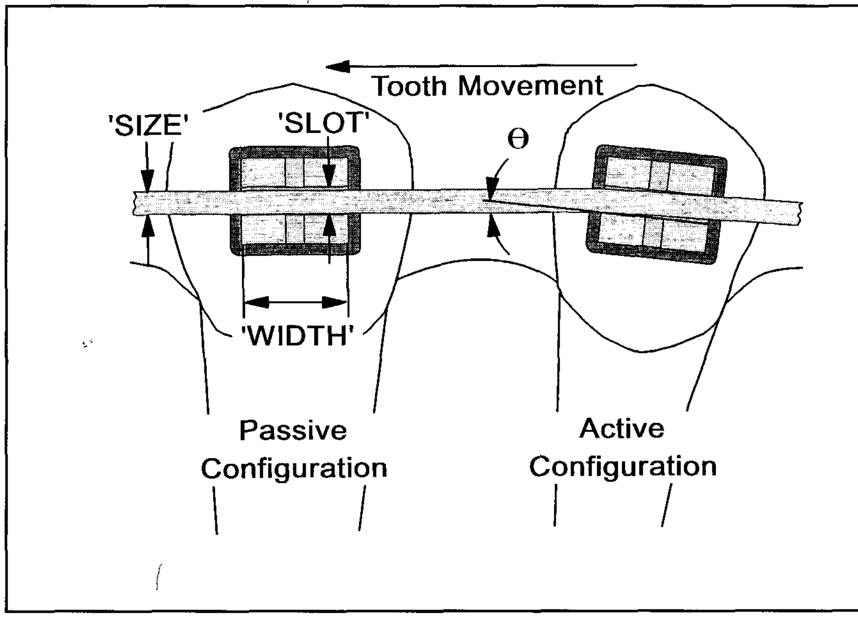

Figure 1
Geometric parameters of importance during sliding mechanics: SIZE of archwire, SLOT and WIDTH of bracket, and contact angle between archwire and bracket (θ). The left-hand tooth illustrates the passive configuration in which second-order clearance exists between opposing tie-wings; the right-hand tooth illustrates the active configuration in which clearance not only no longer exists but θ actually exceeds the critical contact angle for binding, θ_c (not shown, but evidenced by the internal angle of the wire relative to the bracket WIDTH being less than the external θ shown). Note that in this case, the direction of tooth movement is critical, because if the right-hand tooth had been moving in the opposite direction, then θ would equal θ_c.

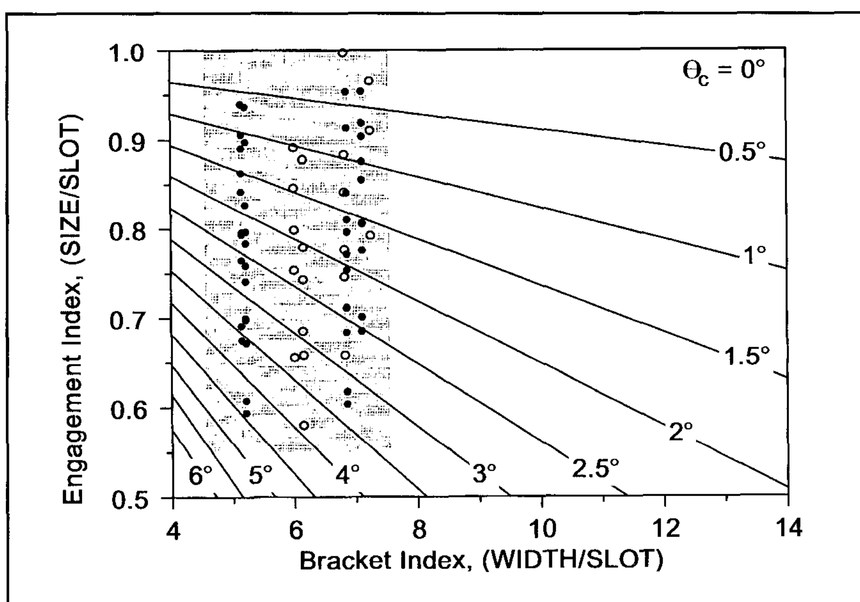

Figure 2
Actual boundaries of commercial products (shaded area) as determined by the minimization-maximization algorithm for the SLOT data (o) and the WIDTH data (•). In all, some 50 archwires and brackets were studied in 64 of the 540 possible combinations to determine that the maximum $\theta_c \leq 5°$, that the (SIZE/SLOT) > 0.55, and that the 4.5 < (WIDTH/SLOT) < 7.5.

maxima of the actual bracket indices, i.e., the actual WIDTH/SLOT ratios (Table 2). Thus, by evaluating only 64 combinations, the actual boundaries of θ_c can be delineated for 540 possible combinations of commercial products (Tables 1 and 2).

Computations

All engagement index (SIZE/SLOT) against bracket index (WIDTH/SLOT) plots and their subsequent calculated θ_c values were obtained via eqns. 1 and 2 or equivalently via eqn. 3.

Experimental verification

As a validation of these calculations, five archwire-bracket couples were selected to represent a range of bracket and engagement indices: a 19x25 SIZE in an 18 SLOT (i.e., a 19x25/18), an 18/18, a 16/18, a 16x22/22, and a 14/22. The first represented the high end of the engagement indices (which, in this case, should have been impossible!), and the last represented the low end of the engagement indices. To avoid confounding the data by varying the alloy, all couples were stainless steel (SS). This decision not only made the stiffness (resistance to elastic deformation) very high but also placed the smoothest surfaces having the lowest frictional coefficient against one another. The efficiency and reproducibility of such couples have been shown to be quite good when clearance exists, i.e., in the passive configuration (Figure 1).[11,12]

Using the same instruments as above, archwire SIZEs were measured at six locations, bracket SLOTS were measured five times on each side for a total of 10 measurements per bracket, and bracket WIDTHS were measured three times at the widest point. Using a special frictional testing device[13,14] in which adjacent brackets represented by frictionless bearings were placed at a great distance (16 mm) from the test bracket, the resistance to sliding (RS) was measured in the dry state at 34°C using a normal force of 300 g. This normal force was chosen for convenience, as θ_c is dependent only on geometry. The RS values of two test runs were made for each of the five aforementioned archwire-bracket couples according to each of the following two schemes. In the first, θ began at 0° by rotating the bracket until its WIDTH dimension was parallel to the long axis of the archwire and then proceeded in 0.5° increments to 5°, in 2° increments from 6° to 12°, and then rotated back to 0°. In the second, θ began at 0° and proceeded in 2° increments from 12° to 6°, and in 0.5° increments from 5° to 0°. Because the RS values of each independent test run were measured at θ = 0° both at the beginning and at the end of each run, the measurements at 0° verified that no change in the bracket surface occurred as each archwire was drawn through its bracket at different values of θ. Al-

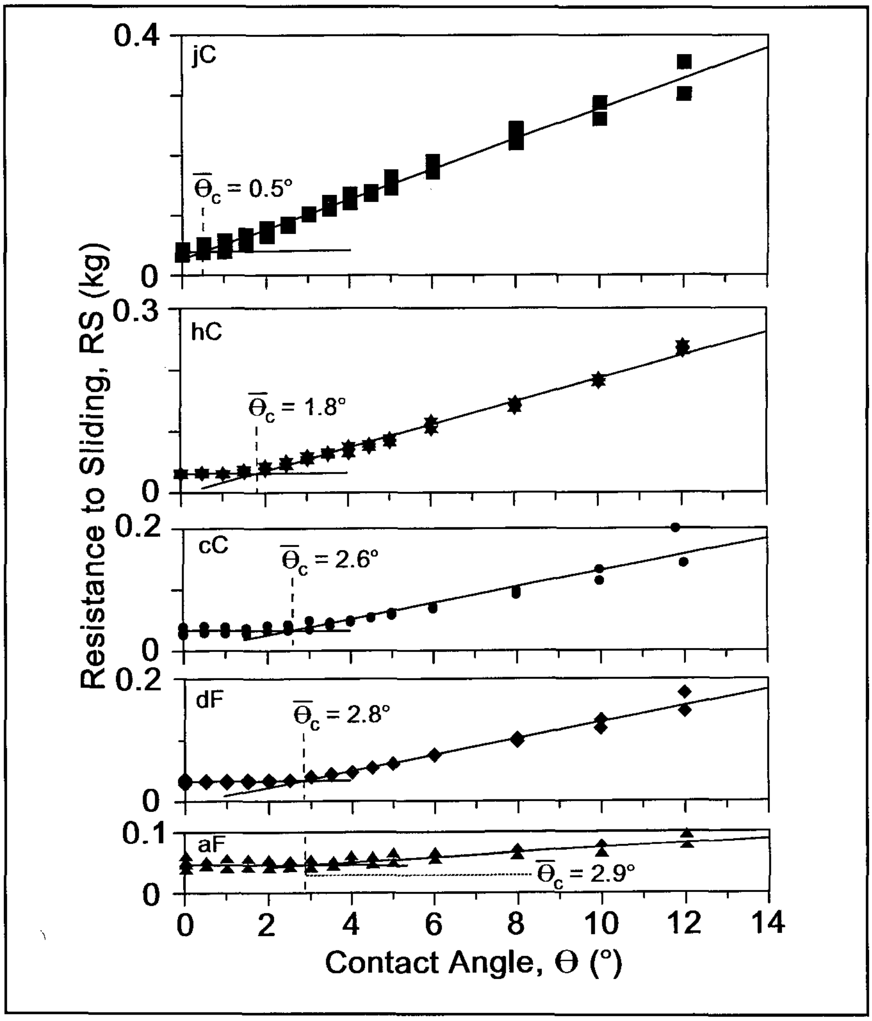

Figure 3
Plot of contact angle (θ) versus resistance to sliding (RS) for five stainless steel on stainless steel (SS-SS) couples in which engagement indices (SIZE/SLOT) and bracket indices (WIDTH/SLOT) differed greatly (Table 3). By inscribing linear regression lines to the upper and lower parts of all θ vs. RS data, the average values of the measured critical contact angle ($\bar{\theta}_c$) were adduced. From actual dimensional measurements and eqns. 1 and 2, the calculated values of θ_c may be compared (Table 3). When tested at a normal force (N) of 300 g in the dry state at 34°C, the average measured θ_c values ($\bar{\theta}_c$) ranged from a low of $\bar{\theta}_c$ = 0.5° for a 19x25/18 SS-SS couple (jC) to a high of $\bar{\theta}_c$ = 2.9° for a 14/22 SS-SS couple (aF).

Table 3
Archwire-bracket couples selected for experimental verification of the calculated critical contact angle for binding (θ_c)

Archwire-bracket couples evaluated	Test run #	Actual SIZE (mils)	Actual SLOT (mils)	Actual WIDTH (mils)	Measured* θ_c (°)	Engagement index, SIZE/SLOT	Bracket index, WIDTH/SLOT	Calculated** θ_c (°)
jC	1	19.10	20.7	98.0	0.3	0.919	4.71	1.0
(19x25/18)	2	19.10	20.5	96.0	0.8	0.931	4.68	0.8
hC	1	18.18	20.1	100.0	1.4	0.904	4.97	1.1
(18/18)	2	18.18	20.1	99.6	2.2	0.902	4.94	1.2
cC	1	15.64	20.4	99.1	2.4	0.765	4.85	2.7
(16/18)	2	15.64	20.0	99.1	2.7	0.781	4.95	2.5
dF	1	16.22	23.5	139.9	2.8	0.689	5.94	3.0
(16x22/22)	2	16.22	23.4	142.2	2.7	0.692	6.07	2.9
aF	1	13.70	23.3	142.1	2.5	0.586	6.08	3.8
(14/22)	2	13.70	23.3	141.0	3.1	0.588	6.05	3.8

* Measured by extrapolating the regression lines of individual θ versus RS plots.
** Calculated via Figure 2, or eqns. 1 and 2, or eqn. 3.

though RS values under both static and kinetic conditions were measured, only the latter were reported because they represented the average of at least 500 data points for each test run. These RS values equaled one-half of the actual measured drawing forces;[12,15] and when clearance exists in the so-called passive configuration (Figure 1), the RS values equaled the frictional forces from which the frictional coefficients could be calculated.

Statistical analyses

For each of the 10 test runs (two for each of the five archwire-bracket couples evaluated), a regression line was computed for each plot of RS against θ, when $\theta < \theta_c$ and when $\theta > \theta_c$. From the intersection of each pair of regression lines, a "Measured θ_c" was obtained. The data of each measured θ_c was plotted against each "Calculated θ_c," and the regression line was determined. This line was compared with a second line, which presumes that the calculated θ_c (i.e., the mathematical relationship) and the measured θ_c (i.e., the experimental determination) are equivalent. When the data of the two RS against θ runs were combined for each couple and the same linear regression analyses were made, an average value of measured θ_c ($\overline{\theta}_c$) was obtained. In all cases, the probability (p) of all regression lines were determined from the correlation coefficient (r) and the number of data points (n).

Results

Examination of archwire dimensions showed that the actual SIZE of archwires can be not only undersized (as expected) but also oversized in 30% of the 26 wires investigated (Table 1). In about 15% of the bracket measurements reported, the SLOTS were smaller than that nominally stated (Table 2). In some of the remaining brackets studied, the SLOTS exceeded the nominal value by as much as 16% and 8% in the cases of nominal 18 and 22 SLOTS, respectively. Of course, bracket WIDTHS are not specified on labels, so no nominal values exist. Suffice to say that the WIDTHS varied by more than 50% as they ranged from 99 mils to 156 mils. In one bracket, F, the WIDTHS actually increased 9 mils from the base of the SLOT to the top of the tie-wings.

The adduced boundaries of commercial products (Figure 2, shaded area) show that θ_c ranged from 0° to about 5°. The corresponding engagement and bracket indices ranged from about 0.55 to 1.0 and from 4.5 to 7.5, respectively. In the final analysis, the minimization-maximization algorithm for the SLOT values delineated the top, right, and bottom boundaries of the bracket index-engagement index plot. Only the left-hand boundary was delineated by the algorithm based upon the WIDTH values.

The regression lines of θ versus RS were highly significant ($p < 0.001$ in Figure 3). For replicate test runs of five archwire-bracket couples, these plots show that, as the SLOT was progressively filled, the value of θ_c occurred earlier; consequently, the binding that ensued was more severe. For example, for the couples aF and jC, a distinctive break occurred between $3.1° > \theta_c > 2.5°$ for the measured θ_c of the 14/22 couple and between $0.8° > \theta_c > 0.3°$ for the measured θ_c of the 19x25/18 couple (Table 3). As the value of θ increased to 12°, the values of RS increased fourfold, from 83 g for aF to 329 g for jC (Figure 3). From calculations of the engagement and bracket indices based on the actual measurements of these specific archwires and brackets (calculated θ_cs in Table 3), Figure 2 (which embodies eqns. 1 and 2 or eqn. 3) indicated that the average θ_c or $\bar{\theta}_c$ = 3.8° for the 14/22 couple and $\bar{\theta}_c$ = 0.9° for the 19x25/18 couple. As Figure 4 shows, the linear regression (solid line) of the calculated versus measured θ_c values were significant (r = 0.839 and n =10 for a *p*-value < 0.01) and generally approximated a 1:1 relationship (dashed line).

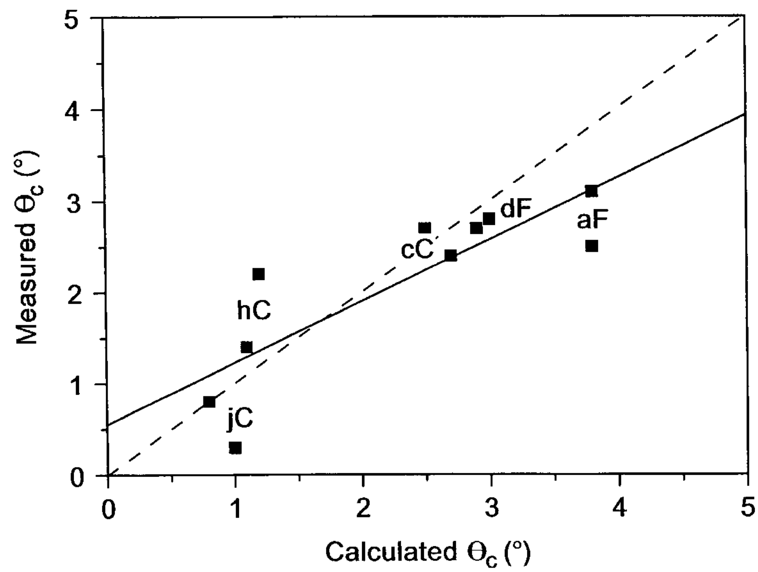

Figure 4
Linear regression of calculated θ_c versus measured θ_c values for replicate test runs of five SS-SS couples at N = 300 g in the dry state at 34°C: jC, hC, cC, dF, and aF (Table 3). From the correlation coefficient and the number of data points evaluated, p < 0.01, thus further substantiating the validity of the earlier derived results.[10] The dashed line (- - -) denotes the 1:1 relationship.

Discussion

Influence of archwire-bracket dimensions

Wires and brackets should be designed so that the wire of one vendor will universally fit into the bracket of another. To accomplish this, wire SIZES should be, in engineering tolerance notation, plus zero and minus some value. To complement the wires, the bracket SLOTS should be minus zero and plus some value. In this way an interference fit would never occur between a wire SIZE of the same dimension as a bracket SLOT, even though the SLOT is filled. Other engineering tolerance schemes will work, too, but for all schemes the dimensions should be such that the mean plus three standard deviations of the archwire SIZE does not exceed the mean minus three standard deviations of the bracket SLOT. Such a standard would provide a 99% confidence level. Clearly this is not being done today, as the SIZES of some wires exceed the nominal SLOT dimensions and the SLOTS of some brackets are smaller than the nominal SLOT dimensions. The SLOTS of the foreign vendors may be somewhat larger than those of the domestic vendors, too, perhaps because they are using metric tooling and targeting 0.5 mm (i.e., 19.7 mils) for the 18 mil SLOT and 0.6 mm (i.e., 23.6 mils) for the 22 mil SLOT (Table 2). For all brackets, WIDTHS must be specified.

From the perspective of sliding mechanics and the present analysis, having a larger SLOT is not necessarily bad, providing that the strengths and stiffnesses of the tie-wings and retention pads remain adequate. As the engagement and bracket indices decrease in magnitude (Figure 2, shaded area), the allowable θ_c increases. All other parameters being equal, increasing the SLOT actually ameliorates binding as the SLOT reduces the engagement and bracket indices. Reducing the SIZE and WIDTH of wires and brackets, respectively, also provides some benefits in increasing the allowable value of θ_c. The wire SIZE can be reduced as long as the strength, resilience, and force per unit of deactivation are sufficient; the bracket WIDTH can be reduced as long as control of each tooth can be maintained. Therefore, these observations—that vendors have not only made larger bracket SLOTS but also have generally made smaller wire SIZES and bracket WIDTHS—facilitates the initiation of sliding mechanics as they obviate the need for as precise initial leveling and alignment.

This can be readily seen in Figure 5 by comparing how the boundaries of commercial products have generally expanded and shifted (the shaded area) relative to the nominal values calculated elsewhere[10] for 3s, 4s, and 5s (the inscribed area). Thus, shifting the nominal dimensions of an archwire-bracket couple from the inscribed area to an actual location that is down and/or to the left (Figure 5) produces a couple that is capable of sliding at a higher θ_c. Based on the nominal values for SIZE, SLOT, and WIDTH (125 mils) of the five couples that were used for experimental verification (Table 3), greater shifts were observed for those wires that were tested against 18 mil than 22 mil brackets. The extreme case was jC, which should be impossible with a nominal engagement index greater than one. The calculated θ_cs of each of these three couples (jC, hC, and cC) increased over 1°. Using the first run of the 16/18 couple (cC) as an example, the engagement and bracket indices for nominal dimensions are 0.889 and 6.94, respectively. These would be plotted in the upper right corner of the inscribed area (Figure 5), and θ_c would equal 0.9°. From the actual dimensions of this couple, the engagement and bracket indices are 0.765 and 4.85, respectively (Table 3), and the calculated θ_c was 2.7° (Table 3). For the couples tested against the calculated 22 mil brackets (dF and aF), θ_c actually increased less than 0.5° more than the nominal dimensions predicted. Nonetheless in all clinical cases, the values of θ_c will never exceed a maximum of about 5°, independent of the available hardware, the wire technique, or the practitioner.

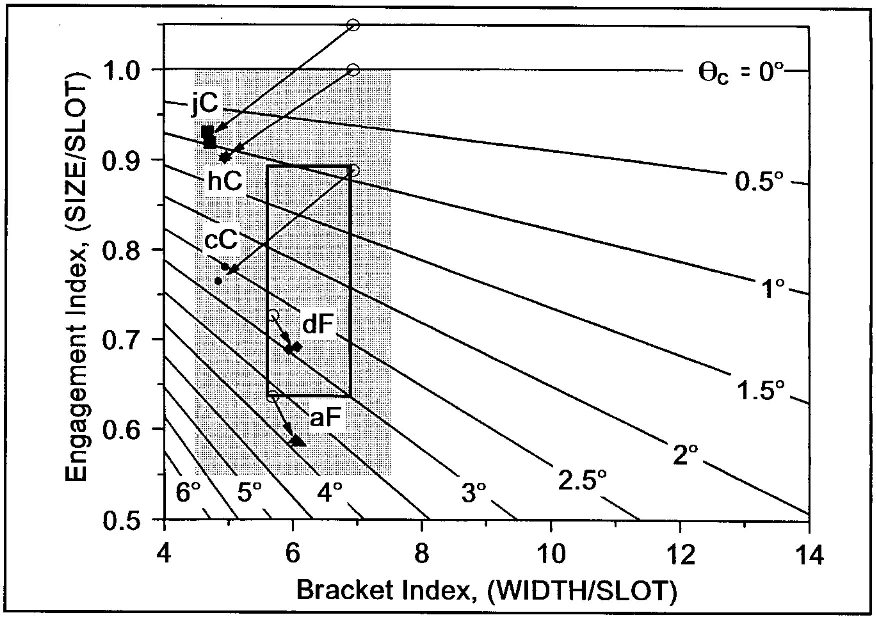

Figure 5
Comparison of the actual boundaries of commercial products (shaded area) with the nominal values that were calculated (inscribed area representing 3s, 4s, and 5s from Figure 3 of ref. 10). In general when the nominal SIZES had been overestimated and nominal SLOTS had been underestimated during manufacturing (Tables 1 and 2) the boundaries shifted down and/or towards the left. When the measured θ_cs of two replicates of five archwire-bracket couples (Table 3) are plotted against this backdrop, the shift of the θ_cs from that of the nominal θ_c (the Os) to that of the measured θ_cs (the arrowheads) increased from less than 0.5° for the 22 mil SLOTS to over 1° for the 18 mil SLOTS.

Presence and absence of second-order clearance

Because θ_c represents the demarcation line between facile sliding on one hand and restricted sliding on the other, the data shown in Figure 3 may be shifted with respect to θ_c. Assuming that the average θ_c or $\overline{\theta}_c$ equals a demarcation point above which relative angles represent the absence of second-order clearance (i.e., the active configuration; Figure 1) and below which relative angles represent the presence of second-order clearance (i.e., the passive configuration; Figure 1), Figure 6 results in two distinctive zones.

When $\theta < \overline{\theta}_c$ and the passive configuration exists (Figure 6), RS is fairly constant and low in magnitude. These values are equivalent to frictional forces previously reported for SS-SS couples in the literature.[12] In this specific case, when N = 300 g in the dry state at 34°C, RS = 35 ± 5 g for a kinetic coefficient of friction (μ_K) = 35 g/300 g = 0.12. Here the wire and bracket geometry have little effect as contact areas are established and adjusted in accordance with the second law of friction.[16,17] Consequently, these five SS-SS couples behave similarly. This experimental outcome confirms that classical friction dominates as binding and its associated phenomena are nonexistent.[16,18,19] Once the θ reduces the clearance of the archwire within the bracket to zero (i.e., $\theta = \overline{\theta}_c$), RS begins to increase because binding begins to contribute to classical friction.

As $\theta > \overline{\theta}_c$ (Figure 6), binding increasingly contributes to the value of RS. Thus, as binding progresses, it overwhelms any contribution from classical friction and approximates the total value of RS. For the same relative contact angle ($\theta - \overline{\theta}_c$) then, RS is greatest (Figure 6) when the engagement index is greatest and the bracket index is smallest (the 19x25/18 SS-SS couple [jC] versus other archwire-bracket couples in Table 3 and Figure 5). Moreover when $\theta >> \overline{\theta}_c$, more serious problems that are associated with binding can occur—namely notching,[16,18,19] which can stop sliding mechanics. From these experimental results then, full engagement carries with it an obligation to have precise initial leveling and alignment, whereas selecting geometric parameters to obtain a high value of θ_c carries with it an obligation to never have $\theta >> \theta_c$. In the first case, sliding would at the very least be impaired by binding; in the latter, sliding would likely cease altogether by notching.

Clinical recommendations

In the previous paragraphs, the values of θ_c have been discussed in terms of their effects on classical friction and binding. There are other considerations that deserve mentioning, foremost of which is the effect θ_c has on clinical practice. By knowing θ_c, treatment time may be decreased by not initially over-aligning teeth prior to using sliding mechanics. In the past, an intuitive concern that θ should never exceed θ_c sometimes prompted clinicians to over-align teeth at the expense of greater chair time. According to Figure 3, the practitioner should initiate sliding mechanics once θ approaches a characteristic value of θ_c (or $\bar{\theta}_c$) for the particular archwire-bracket couple of choice. By ensuring that θ ≈ θ_c, the optimal solution is attained by minimizing the financial ramifications associated with either increased chair time due to over-alignment (when $\theta < \theta_c$) or increasingly sluggish or nonexistent sliding mechanics (when $\theta > \theta_c$). As Figure 6 shows, for $\bar{\theta}_c$, no advantage exists in aligning teeth with more tooth-to-tooth precision than $\theta = \bar{\theta}_c$, but real and severe penalties can be associated with attempting to use sliding mechanics when $\theta > \bar{\theta}_c$, particularly as the SLOT is filled (e.g., the 19x25/18 SS-SS couple [jC]). Regardless of wire-bracket geometry, the optimal clinical value of θ should *approach* the characteristic value of θ_c for that particular wire-bracket couple, any exceptions occurring only when the SLOT is not filled (e.g., the 14/22 SS-SS couple [aF]). Other parameters, such as elastic modulus, yield strength, hardness, and surface roughness, may also influence the coordinates of (θ, RS) during sliding, but they will be considered in future reports.

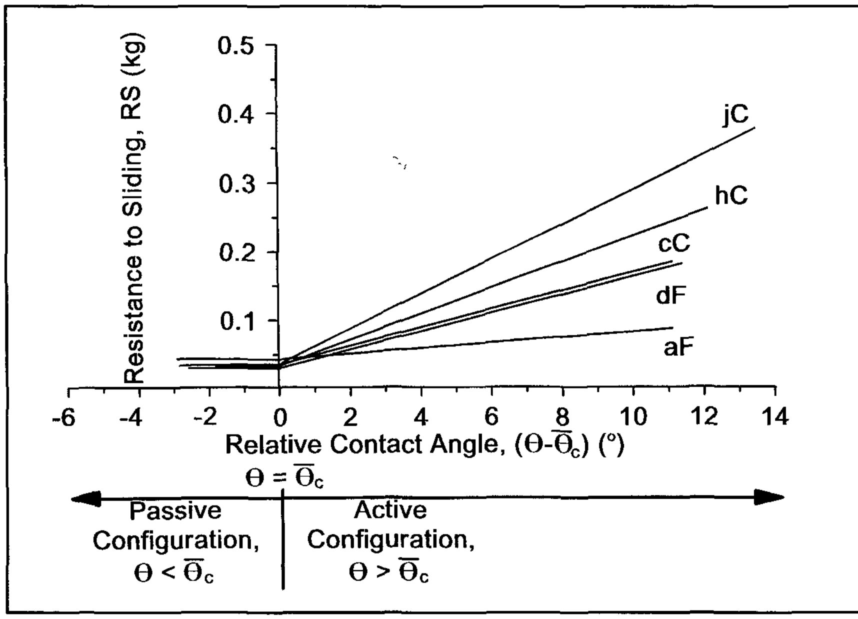

Figure 6
Regression lines from replicate test runs of five SS-SS couples, having been tested at N = 300 g in the dry state at 34°C (Figure 3). Only here, each regression line was horizontally shifted by $\bar{\theta}_c$ so that each $(\theta - \bar{\theta}_c)$ versus RS plot would superpose at $\theta = \bar{\theta}_c$. Thereby the passive configuration (wherein clearance exists and only classical friction occurred; Figure 1) was illustrated to the left of $(\theta - \bar{\theta}_c) = 0$, and the active configuration (wherein no clearance exists and binding rapidly dominated; Figure 1) was illustrated to the right of $(\theta - \bar{\theta}_c) = 0$. Note that as the engagement index increased and the bracket index decreased from the 14/22 SS-SS couple (aF) to the 19x25/18 SS-SS couple (jC) (Table 3), binding increased as indicated by the slopes of the $(\theta - \bar{\theta}_c)$ versus RS lines. In other words, in the absence of notching phenomena, archwire-bracket couples having less second-order clearance ultimately bind more severely than those couples having more second-order clearance.

Conclusions

When a minimization-maximization algorithm was applied to 50 archwires or brackets, the maximum value of the critical contact angle for binding (θ_c) compared favorably with the nominal wire-bracket calculations that were made using derived equations. Overall, actual SIZES of archwires are smaller than their nominal SIZES, and actual SLOTS of brackets are larger than their nominal SLOTS, ultimately driving the aforementioned indices toward the maximum value of $\theta_c \approx 5°$.

Experimental verification of θ_c via contact angle (θ) versus resistance to sliding (RS) measurements for five stainless steel on stainless steel (SS-SS) couples validated the calculated results for θ_c. These couples also showed how invariant classical friction is for SS-SS couples at $\theta < \theta_c$ and how rapidly binding can outstrip classical friction once $\theta > \theta_c$.

Clinical practitioners derive no tangible benefits from $\theta < \theta_c$ or $\theta > \theta_c$. In the former, too precise alignment equates to excessive chair and total treatment time; in the latter, sliding declines due to binding or may stop altogether due to notching of the archwire.

Variable SIZES of archwires and SLOTS and WIDTHS of brackets can result in engagement and bracket indices that differ from those expected. Therefore, geometry must be clearly specified (SIZE, SLOT, and WIDTH); otherwise, θ_c may be miscalculated and sliding compromised.

Acknowledgments

We wish to thank the various vendors for the wires and brackets used in this investigation and Mr. Landon Poteat for making some of the archwire and bracket measurements cataloged in Tables 1 and 2.

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